Vol. XLIX. OCTOBER, 1923. No. 8.

AMERICAN SOCIETY OF CIVIL ENGINEERS

INSTITUTED 1852

REPORTS, PAPERS, DISCUSSIONS, AND MEMOIRS

This Society is not responsible for any statement made or opinion expressed in its publications.

CONTENTS

Papers: PAGE

Reactions for a Particular Type of Unsymmetrical Arch.
By CARL B. ANDREWS, ASSOC. M. AM. SOC. C. E. 1685

Periodic Fluctuations of Rainfall in Hawaii.
By JOEL B. COX, ASSOC. M. AM. SOC. C. E. 1699

The River and Harbor Problems of the Lower Mississippi: A Symposium:
The Mouths of the Mississippi River.
By ELLIOTT J. DENT, M. AM. SOC. C. E. 1722

Reports of Special Committees:

Progress Report of the Special Committee to Codify Present Practice on the Bearing Value of Soils for Foundations, etc. 1732

Discussions:

The Design of Earth Dams.
By MESSRS. THADDEUS MERRIMAN and JOHN G. HIRSCH 1744

Theoretical Frequency Curves and Their Application.
By MESSRS. L. STANDISH HALL, and R. D. GOODRICH 1751

The Disintegration of Cement in Sea Water.
By MESSRS. B. JEANNERET, A. H. RHETT, I. HIROI, and THEODORE BELZNER. 1762

Improved Type of Multiple-Arch Dams.
By MESSRS. B. F. JAKOBSEN, VICTOR H. COCHRANE, and WILLIAM CAIN 1767

Stresses in Multiple-Arch Dams.
By WILLIAM CAIN, M. AM. SOC. C. E. 1785

The River and Harbor Problems of the Lower Mississippi: A Symposium.
By MESSRS. W. M. BLACK, H. B. MUCKLESTON, H. C. RIPLEY, and H. F. DUNHAM. 1792

Railroad Transportation and Railroad Terminals: A Symposium.
By CLEMENT C. WILLIAMS, M. AM. SOC. C. E. 1799

Memoirs:

HENRY EDWARD COANE, M. AM. SOC. C. E. 1806
ROBERT BREWSTER STANTON, M. AM. SOC. C. E. 1807
CLARENCE EVERETT TAIT, M. AM. SOC. C. E. 1809

PLATES

Plate VII. Sections Showing Lines of Mean Temperature, Total Precipitation, and Prevailing Winds in Hawaii, in 1920 1703

For Index to all Papers, the discussion of which is current in *Proceedings*, see the second page of the cover.

REACTIONS FOR A PARTICULAR TYPE OF UNSYMMETRICAL ARCH

By Carl B. Andrews,* Assoc. M. Am. Soc. C. E.

Synopsis

If an unsymmetrical arch is of such form that its axis may be obtained by any number of parallel projections from the axis of a symmetrical arch, and the ratio of the length of each element of the axis to the moment of inertia of a right section of the arch rib, at the middle point of such element, is the same in both arches, the statically indeterminate reactions for unit loads on the unsymmetrical structure may be obtained with comparative ease from the corresponding reactions for the symmetrical one, when the effect of the axial stress is neglected. If the effect of the axial stress is to be considered, there is no simple relation between the reactions for the two types of structure.

The first part of this paper is a discussion of the simple case in which the series of parallel projections is reduced to a shearing distortion; the second part gives the derivation of the formulas referred to.

Part I

As the masonry arch with fixed ends is a statically indeterminate structure, its reactions are to be computed only by the use of the equations obtained from a consideration of elastic distortions. The work of determining the reactions for any given loading is generally long. If the arch is symmetrical, the work is shortened considerably.

The nomenclature herein is similar to that used by M. A. Howe, M. Am. Soc. C. E., in his book, "Symmetrical Masonry Arches", in which there is given the development of the formulas for the reactions of a symmetrical arch.

In arch analysis, there is required a summation of certain quantities, from one end of the arch to the other, and as the shapes of arch axes are not always such as to be summed easily by integration, it is convenient to consider the arch as divided into a finite number of separate parts, and to make the summation in the ordinary way. In most cases, if the arch is divided into twenty such parts, the error due to summation of finite quantities instead of summation by integration, is negligible.

In what follows, the arch span is divided into twenty equal parts, each of length, δx; these parts are projected on the axis, thus dividing it into twenty parts, the length of each part being δs. These parts are of unequal lengths,

Note.—Written discussion on this paper, which will not be presented at any meeting of the Society, will be closed with the January, 1924, *Proceedings*. When finally closed, the paper, with discussion, will be published in *Transactions*.

* Prof. of Eng., Univ. of Hawaii, Honolulu, Hawaii.

although, if the arch is symmetrical, for each segment, δs, on one side of the center line, there is a similar one on the other side. Let the middle points of these segments, δs, be considered as the points at which loads are applied, and let them be called Point 1, Point 2, etc., starting at the left end of the arch and increasing to Point 10 next to the center line on the left side; let the corresponding points on the right side have corresponding numbers, 1′, 2′, etc. As an approximation, for the sake of ease of computation, let the radial cross-section of the arch rib be considered as constant over each length of segment, δs, the cross-section being that which occurs at the middle point of the segment under consideration. Let b = the breadth of such a cross-section, d = the depth, I = its moment of inertia, and F = the area.

Let δy be the increment of vertical ordinate for each segment of the arch axis; for each δx there is a corresponding δy, and the quotient, $\delta x \div \delta y$, will express the tangent of the angle of slope of the axis at the middle point of the segment. In the symmetrical arch, the values of δy are symmetrical on each side of the center line, whereas in an unsymmetrical arch, this is not true.

Further, let,

H_1 = the horizontal thrust (reaction) at the left support produced by vertical loading;

H = the horizontal thrust (reaction) at the left support produced by vertical loading, when the effect of axial stress is neglected;

M_1 = the bending moment at the left support;

M_2 = the bending moment at the right support;

M_x = the bending moment at a point in the arch axis distant x from the left support;

m_x = the bending moment which would occur in a simply supported beam of the same span as the arch, at a distance, x, from the left support;

N_x = the normal component of the resultant force acting on a radial section distant x from the left support, that is, the axial thrust;

l = the span of the arch axis (the horizontal distance between ends of the arch axis); and,

h = the difference in elevation between the ends of the arch axis, in the case of the unsymmetrical arch.

With the left end of the arch axis as the origin of the co-ordinates, take x and y as the co-ordinates of any point on the axis of a symmetrical arch, and x' and y' the co-ordinates of any point on the axis of an unsymmetrical arch.

When the values, δs and I, refer to the same arch segment, let the ratio, $\frac{\delta s}{I}$ be designated by Δ.

Consider the arch as subjected to unit loads at each of the several points of loading, separately; the corresponding values of H_1, M_1, and M_2 will then be

coefficients by which the actual loads at the various points may be multiplied, in order to obtain the actual reactions.

With the exception of the symbols which refer to an unsymmetrical arch, the nomenclature as given herein is the same as that used by Professor Howe. (See Fig. 1.)

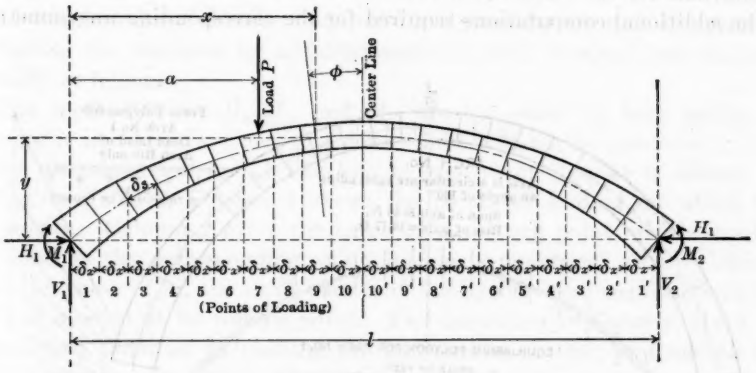


FIG. 1.

If an unsymmetrical arch axis is obtained by a supposed shearing distortion of a symmetrical arch axis, and the value of the ratio, $\Delta \left(= \frac{\delta\, s}{I} \right)$ for each segment of the unsymmetrical arch is the same as that for the corresponding segment of the symmetrical arch, it will be shown that for unit loads at the various points of loading (or for equal loads on the two arches), and neglecting the effect of axial stress,

H, the horizontal thrust, will be the same for the symmetrical and the unsymmetrical arch;

M_1 or M_2, the bending moment at the left or right abutment, respectively, will be the same for the symmetrical and the unsymmetrical arch;

V_1, the vertical reaction at the left abutment, in the case of the unsymmetrical arch, will be equal to the corresponding reaction in the case of the symmetrical arch, plus the product, $H \times \frac{h}{l}$.

The proof of these statements is to be found in the demonstration in Part II.

In order to exhibit the changes which will be brought about in the reactions by such a vertical shearing distortion of a symmetrical arch (neglecting the effect of axial stress), the writer prepared equilibrium polygons for six arches, four of the arches being originally of circular segmental form, one parabolic, and one semi-elliptical. The span of each of these arches was 40 ft., and the unsymmetrical arch was obtained from the symmetrical one by raising the right end 5 ft. above the left end. The vertical ordinates

of points on the arch axis, measured from the chord, are the same in the original arch as in the distorted arch. In all the arches the value of $\Delta \left(= \frac{\delta s}{I}\right)$ has been taken arbitrarily as 3 for all segments of the arch.

In order to show the labor involved in obtaining the reactions, the full computations for one of the symmetrical arches (the arch indicated in Fig. 2), and the additional computations required for the corresponding unsymmetrical

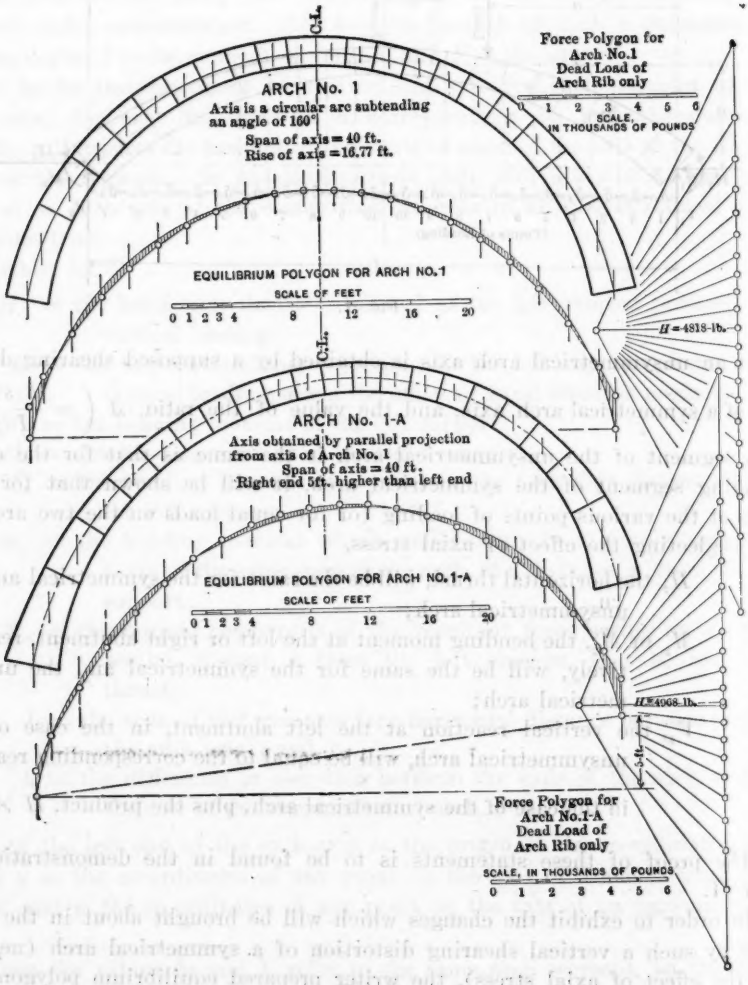

C.L.
ARCH No. 1
Axis is a circular arc subtending an angle of 160°
Span of axis = 40 ft.
Rise of axis = 16.77 ft.
EQUILIBRIUM POLYGON FOR ARCH NO. 1
SCALE OF FEET
0 1 2 3 4 8 12 16 20
C.L.
ARCH No. 1-A
Axis obtained by parallel projection from axis of Arch No. 1
Span of axis = 40 ft.
Right end 5ft. higher than left end
EQUILIBRIUM POLYGON FOR ARCH NO. 1-A
SCALE OF FEET
0 1 2 3 4 8 12 16 20
Force Polygon for Arch No. 1 Dead Load of Arch Rib only
0 1 2 3 4 5 6
SCALE, IN THOUSANDS OF POUNDS
H = 4818 lb.
H = 4008 lb.
5 ft.
Force Polygon for Arch No. 1-A Dead Load of Arch Rib only
SCALE, IN THOUSANDS OF POUNDS
0 1 2 3 4 5 6

FIG. 2.

arch, are given in Tables 1, 2, and 3, these computations having been made in the general manner indicated by Professor Howe, in "Symmetrical Masonry Arches."

For purposes of comparison, the load on each of these arches has been taken as the weight of the arch rib itself, with no superimposed load, which would never be the case in a bridge structure. The equilibrium polygons, therefore (Fig. 2), are not necessarily to be considered as giving an indication of the most favorable shape for a structure.

If it is assumed that the reaction coefficients, that is, the reactions for unit loads, have been already computed for a symmetrical arch, the procedure for finding the reactions for an unsymmetrical arch, obtained from the other, is briefly as follows:

The coefficients for H, M_1, and M_2, are the same for both arches, but, as there is no longer symmetry of form, there will not be symmetry of loads on the unsymmetrical arch. The load at each point is to be tabulated. Referring to Table 3, Column 1 shows the points of loading, of which there are twenty. Column 2 gives the value of Δ at each point, which should be the same as the corresponding value for the symmetrical arch. Column 3 gives the value of δs, the axial length of the hypothetical segment which has the load applied at its middle point. The quantities in Column 3 are most conveniently obtained by platting the unsymmetrical arch, and scaling these various lengths. In Column 4, the moment of inertia, I, is given, which is obtained as the quotient of the respective quantities in Columns 3 and 2. As the breadth of the arch rib under consideration is taken for convenience as 1 ft. (or unity), the values of d^3 in Column 5 are obtained by multiplying the values of I by 12, and from the values of d^3, the values of d, the depth of the arch rib, are obtained, which are placed in Column 6. With these values of d, the outline of the unsymmetrical arch is to be platted, and from the plat the volume of material, the weight of which is to be considered as concentrated at the loading point, may be obtained by scaling or by planimeter. Any small error arising from the graphical methods will probably be less than the unknown error likely to occur through inaccuracy of forms or falsework. From the values in Column 7, those in Column 8 are obtained. Column 9 serves as a check on the arithmetic. The values in Columns 10, 11, and 12 are obtained, respectively, by multiplying the numbers in Column 8 by the coefficients for H, M_1, and M_2, as obtained for the symmetrical arch. Columns 13 and 14 lead to the value of $\Sigma_o^l P \frac{l-a}{l}$, which is required for finding the value of V_1. V_2 will be the difference between the total load and V_1. When V_1 and V_2 are known, the force polygon and the equilibrium polygon may be platted.

The positions of maximum moment will be indicated by the equilibrium polygon, and if it is desired to know the maximum moments accurately, they may be computed by the usual statical equations.

When the effect of axial stress is considered, it is found that there is no simple relation between the horizontal reactions of the symmetrical and the unsymmetrical arches, because some terms in the expression for the horizontal reaction reduce to zero with the symmetrical arch, which do not do so in the case of the unsymmetrical arch. The effect of the axial stress is to shorten

TABLE 1.—COMPUTATION OF COEFFICIENTS FOR REPRESENTATIVE ARCHES. ARCH No. 1.

The Axis of Arch No. 1 is a Circular Segment: Span, 40 ft.: Central Angle, 160°

GENERAL DATA AND COMPUTATIONS FOR H_1.

Pt.	x.	y.	Δ.	$y\,\Delta$.	$y-\frac{\Sigma y\Delta}{\Sigma\Delta}$	$B=\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$. (1)	z or k. (2) or (3)	zB (4)	$x=a$ ΣzB $x=0$. (5)	$x=\frac{l}{2}$ ΣB $x=a$. (6)	$x=\frac{l}{2}$ $k\Sigma B$ $x=a$. (7)	Column (5) + Column (7) (8)	$\left(\frac{\delta x}{2\Sigma yB}\right)$ (9)
1	1	3.62	3	10.86	− 9.031	− 27.093	− 98.07666	1	− 27.093	− 27.093	+ 27.093	+ 27.093	0.000	0.0000
2	3	7.55	3	22.65	− 5.101	− 15.303	− 115.53765	3	− 45.909	− 73.002	+ 42.396	+ 127.188	+ 54.186	0.0526
3	5	10.13	3	30.39	− 2.521	− 7.563	− 76.61319	5	− 37.815	− 110.817	+ 49.959	+ 249.795	+ 138.978	0.1349
4	7	12.06	3	36.18	− 0.591	− 1.773	− 21.38238	7	− 12.411	− 123.228	+ 51.732	+ 362.124	+ 238.896	0.2319
5	9	13.52	3	40.56	+ 0.869	+ 2.607	+ 35.24664	9	+ 23.463	− 99.765	+ 49.125	+ 442.125	+ 342.360	0.3324
6	11	14.66	3	43.98	+ 2.009	+ 6.027	+ 88.35582	11	+ 66.297	− 33.468	+ 43.098	+ 474.078	+ 440.610	0.4288
7	13	15.53	3	46.59	+ 2.879	+ 8.637	+ 134.13961	13	+ 112.281	+ 78.813	+ 34.461	+ 447.993	+ 526.806	0.5115
8	15	16.14	3	48.42	+ 3.489	+ 10.467	+ 168.93738	15	+ 157.005	+ 235.818	+ 23.994	+ 359.910	+ 595.728	0.5784
9	17	16.55	3	49.65	+ 3.899	+ 11.697	+ 193.58535	17	+ 198.849	+ 434.667	+ 12.297	+ 209.049	+ 643.716	0.6250
10	19	16.75	3	50.25	+ 4.099	+ 12.297	+ 205.97475	19	+ 233.643	+ 668.310	0.000	0.000	+ 668.310	0.6489
			30	379.53		− 51.732	− 311.60988		− 123.228					3.5444
			2	2		+ 51.732	+ 826.23255		+ 791.538					2
			$\Sigma\Delta=60$	$759.06=\Sigma y\Delta$			+ 514.62267		+ 668.310					$\Sigma H_1=7.0888$
			$\frac{\Sigma y\Delta}{\Sigma\Delta}=\frac{759.06}{60}=12.651$				2		2					
							+ 1 029.24534 $=\Sigma yB$		+ 1 336.620 $=\Sigma zB$					

TABLE 1.—(*Continued*).

TABLE 1.—(*Continued*).

COMPUTATIONS FOR $\frac{\Sigma m_x \Delta}{\Sigma \Delta}$, ETC.

Pt.	z. (1)	$z\,\Delta$. (2)	$x=a$ $\Sigma z \Delta$ $x=0$. (3)	$x=\frac{l}{2}$ $\Sigma \Delta$ $x=a$. (4)	k. (5)	$x=\frac{l}{2}$ $k\,\Sigma \Delta$ $x=a$. (6)	$\Sigma m x \Delta$ $\delta \frac{x}{2}$ $=(3)+(6)$ (7)	$\frac{\Sigma m x \Delta}{\Sigma \Delta}$ $=(7)\cdot\frac{\delta x}{2\,\Sigma \Delta}$ (8)	$n-k$. (1)	k (2)	$z-n$ (3)	z (4)	$(z-n)z$ (5)	$(x-n)z\,\Delta$ (6)	$\Sigma \Delta (z-n) z$ $x=0$ (7)	$x=a$ $\Sigma \Delta (z-n) z$ $x=0$ $(8)-(k)$ (8)	$-(z-n)^2$ (9)	$-\Delta (z-n)^2$ (10)	$\frac{l}{2}$ $-\Sigma \Delta (z-n)^2$ $x=a$ (11)	$\frac{l}{2}$ $-k\,\Sigma \Delta (z-n)^2$ $x=a$ (12)	Column (8)+(12) (13)	$(\frac{\delta x}{2})(-0.000134)\,x$ (14)
1	1	3	3	27	1	27	30	0.50	19	1	−19	1	−19	− 57	− 57	− 1 083	−361	−1 083	−2 907	−2 907	− 3 990	+0.534660
2	3	9	12	24	3	72	84	1.40	17	3	−17	3	−51	−153	− 210	− 3 570	−289	− 867	−2 040	−6 120	− 9 690	+1.29846
3	5	15	27	21	5	105	132	2.20	15	5	−15	5	−75	−225	− 435	− 6 525	−225	− 675	−1 365	−6 825	−13 350	+1.78756
4	7	21	48	18	7	126	174	2.90	13	7	−13	7	−91	−273	− 708	− 9 204	−169	− 507	− 858	−6 006	−15 210	+2.03814
5	9	27	75	15	9	135	210	3.50	11	9	−11	9	−99	−297	−1 005	−11 055	−121	− 363	− 495	−4 455	−15 510	+2.07834
6	11	33	108	12	11	132	240	4.00	9	11	− 9	11	−99	−297	−1 302	−11 718	− 81	− 243	− 252	−2 772	−14 490	+1.94166
7	13	39	147	9	13	117	264	4.40	7	13	− 7	13	−91	−273	−1 575	−11 025	− 49	− 147	− 105	−1 365	−12 390	+1.66026
8	15	45	192	6	15	90	282	4.70	5	15	− 5	15	−75	−225	−1 800	− 9 000	− 25	− 75	− 30	− 450	− 9 450	+1.26680
9	17	51	243	3	17	51	294	4.90	3	17	− 3	17	−51	−153	−1 953	− 5 859	− 9	− 27	− 3	− 51	− 5 910	+0.79194
10	19	57	300	0	19	0	300	5.00	1	19	− 1	19	−19	− 57	−2 010	− 2 010	− 1	− 3	0	0	− 2 010	+0.26934

Pt.	Symmetrical values of z^2, added.
1	1 522
2	1 878
3	1 250
4	1 138
5	1 042
6	962
7	898
8	850
9	818
10	802
	10 660
	3 (= Δ)
	31 980 = $\Sigma z^2 \Delta$

$\Sigma z^2 \Delta = 31\,980.$ $\quad n \Sigma \Delta = 20 \times 60 = 1\,200.$

$$\frac{\Sigma z^2 \Delta}{n \Sigma \Delta} = 26.23 \qquad n - \frac{\Sigma z^2 \Delta}{n \Sigma \Delta} = -6.23$$

$$\left(n - \frac{\Sigma z^2 \Delta}{n \Sigma \Delta}\right) \Sigma \Delta = (-6.23)\,(60) = -373.80$$

$$n\left(n - \frac{\Sigma z^2 \Delta}{n \Sigma \Delta}\right) \Sigma \Delta = (20)\,(-373.80) = -7\,476$$

$$\frac{\frac{\delta x}{2}}{n\left(n - \frac{\Sigma z^2 \Delta}{n \Sigma \Delta}\right) \Sigma \Delta} = \frac{1}{-7\,476} = -0.000134$$

TABLE 2.

Pt.	$H_1 \frac{\Sigma y \Delta}{\Delta \Sigma} = 12.651\, H_1.$ (1)	$\frac{\Sigma mx \Delta}{\Sigma \Delta}$ (2)	Column (14) of Table 1. (3)	Column (2) + Column (3). (4)	Column (2) − Column (3). (5)	M_1 Column (1) − Column (4). (6)	M_2 Column (1) − Column (5). (7)
1	0.0000	0.50	0.5347	1.0347	− .0347	− 1.0347	0. (approx)
2	0.6654	1.40	1.2985	2.6985	+ .1015	− 2.0331	+ 0.5639
3	1.7066	2.20	1.7876	3.9876	+ .4124	− 2.2810	+ 1.2942
4	2.9350	2.90	2.0381	4.9381	+ .8619	− 2.0031	+ 2.0731
5	4.2001	3.50	2.0783	5.5783	+ 1.4217	− 1.3782	+ 2.7784
6	5.4273	4.00	1.9417	5.9417	+ 2.0583	− 0.5144	+ 3.3690
7	6.4773	4.40	1.6603	6.0603	+ 2.7397	+ 0.4170	+ 3.7376
8	7.3123	4.70	1.2663	5.9663	+ 3.4337	+ 1.3460	+ 3.8786
9	7.9069	4.90	0.7919	5.6919	+ 4.1081	+ 2.2150	+ 3.7988
10	8.2105	5.00	0.2693	5.2693	+ 4.7807	+ 2.9412	+ 3.4798

Dead-Load Reactions, Arch No. 1.

Pt.	Coefficients. H	Coefficients. M_1.	Coefficients. M_2.	Weight of concrete, in pounds. ($= P$)	Actual H_1.	Actual M_1.	Actual M_2.	$\frac{l}{2} - a$	$P\left(\frac{l}{2} - a.\right)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	0.0000	− 1.0347	0.0000	2 520	0.0	− 2 607	0	19	47 880
2	0.0526	− 2.0331	+ 0.5639	1 440	75.7	− 2 928	+ 812	17	24 480
3	0.1349	− 2.2810	+ 1.2942	1 050	141.8	− 2 394	+ 1 354	15	17 250
4	0.2319	− 2.0031	+ 2.0731	810	187.9	− 1 620	+ 1 677	13	10 530
5	0.3324	− 1.3782	+ 2.7784	750	249.0	− 1 035	+ 2 085	11	8 250
6	0.4288	− 0.5144	+ 3.3690	720	313.2	− 375	+ 2 460	9	6 480
7	0.5115	+ 0.4170	+ 3.7376	645	330.2	+ 269	+ 2 412	7	4 515
8	0.5784	+ 1.3460	+ 3.8786	600	346.8	+ 810	+ 2 328	5	3 000
9	0.6250	+ 2.2150	+ 3.7988	600	375.0	+ 1 332	+ 2 280	3	1 800
10	0.6489	+ 2.9412	+ 3.4798	600	389.4	+ 1 764	+ 2 088	1	600
	3.5444				2 409.0	− 10 959	+ 17 496		124 785 =
					2	+ 4 175	− 6 784		$\Sigma P\left(\frac{l}{2} - a.\right)$
					4 818.0	− 6 784	− 10 712		
					$= H.$		$= M_1 = M_2$		

$$y_1 = \frac{M_1}{H} = \frac{10\,712}{4\,818} = 2.22 \text{ ft.}$$

$$Mc = M_1 + V_1 \frac{l}{2} - Hf - \Sigma P\left(\frac{l}{2} - a.\right)$$

$$= + 10\,712 + 9\,735\,(20) - 4\,818\,(16.77) - 124\,785$$

$$= + 10\,712 + 194\,700 - 80\,798 - 124\,785 = 205\,412 - 205\,583 = - 171 \text{ lb-ft.}$$

$$yc = - \frac{171}{4\,818} = - 0.0355 \text{ ft.}$$

TABLE 3.—Let Arch No. 1 have its Right End Displaced Upward 5 ft., to be Arch No. 14. The Values of H and M Will not be Changed.

TABLE 3.—LET ARCH NO. 1 HAVE ITS RIGHT END DISPLACED UPWARD 5 FT., TO BE ARCH NO. 1A. THE VALUES OF H AND M WILL NOT BE CHANGED.

For Arch No. 1A, the Value of V_1 will be $V_1 = \frac{M_2 - M_1}{l} + \frac{H_1 h}{l} + \sum_o^l P \frac{l-a}{l}$

Pt.	Δ	δs, in feet.	$= \frac{I}{\delta s}$	$d^3 = 12\,I$	d	Volume concrete (by planimeter), in cubic feet.	Weight concrete $= P$, in pounds	$\Sigma_0^a P$.	Actual H	Actual M_1	Actual M_2	$\frac{l-a}{l}$	$P \frac{l-a}{l}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	3	6.4	2.133	25.596	2.95	20.0	3 000	3 000	0	− 3 104	0	0.975	2 925
2	3	3.95	1.317	15.804	2.51	10.0	1 500	4 500	79	− 3 497	+ 846	0.925	1 388
3	3	3.25	1.083	12.996	2.35	7.5	1 125	5 625	152	− 2 566	+ 1 456	0.875	984
4	3	2.7	0.900	10.800	2.21	6.4	960	6 585	223	− 1 950	+ 1 990	0.825	792
5	3	2.55	0.850	10.200	2.17	5.7	855	7 440	284	− 1 178	+ 2 376	0.775	663
6	3	2.35	0.783	9.396	2.11	5.5	825	8 265	354	− 424	+ 2 779	0.725	598
7	3	2.2	0.740	8.880	2.07	4.7	705	8 970	361	+ 294	+ 2 635	0.675	476
8	3	2.15	0.717	8.604	2.05	4 4	660	9 630	382	+ 888	+ 2 560	0.625	413
9	3	2.05	0.683	8.196	2.02	4.3	645	10 275	403	+ 1 429	+ 2 450	0.575	371
10	3	2.00	0.667	8.000	2.00	4.0	600	10 875	389	+ 1 765	+ 2 088	0.525	315
10′	3	2.00	0.667	8.000	2.00	4.0	600	11 475	389	+ 2 088	+ 1 765	0.475	285
9′	3	2.00	0.667	8.000	2.00	4.0	600	12 075	375	+ 2 279	+ 1 329	0.425	255
8′	3	2.05	0.683	8.196	2.02	4.0	600	12 675	347	+ 2 327	+ 808	0.375	225
7′	3	2.07	0.690	8.280	2.03	4.2	630	13 305	322	+ 2 355	+ 263	0.325	205
6′	3	2.10	0.700	8.400	2.04	4.3	645	13 950	277	+ 2 173	− 332	0.275	177
5′	3	2.20	0.733	8.796	2.06	4.9	735	14 685	244	+ 2 042	− 1 013	0.225	165
4′	3	2.45	0.817	9.804	2.14	5.2	780	15 465	181	+ 2 130	− 1 584	0.175	137
3′	3	2.90	0.967	11.604	2.26	6.65	998	16 463	135	+ 1 292	− 2 276	0.125	125
2′	3	3.60	1.200	14.400	2.43	9.0	350	17 813	71	+ 761	− 2 745	0.075	101
1′	3	5.9	1.967	23.604	2.87	17.2	2 580	20 393	0	0	− 2 670	0.025	65
							20 393		4 968 = H	−12 719 +21 823	+23 345 −10 620		10 665
										+ 9 104 = M_1	+12 725 = M_2		$= \Sigma_o^l P \frac{l-a}{l}$

To obtain V_1: $\frac{M_2 - M_1}{l} = \frac{3\,621}{40} = 91$

$\frac{H_1 h}{l} = \frac{4\,968 \times 5}{40} = 621$

Then, $V_1 = \frac{M_2 - M_1}{l} + \frac{H_1 h}{l} + \Sigma_0^l P \frac{l-a}{l} = 91 + 621 + 10\,665 = 11\,377$ lb

Hence, $V_2 = 20\,393 - 11\,377 = 9\,016$ lb.

The ordinates at the ends of the equilibrium polygon are:

$y_1 = \frac{9\,104}{4\,968} = +1.83$ ft.; $y_2 = \frac{12\,725}{4\,968} = +2.56$ ft.

The coefficients for the reactions, that is, the reactions for unit loads at the different points of loading, having been computed for a symmetrical arch, this table indicates all the work necessary to find the reactions for a distorted arch, which is obtained by supposing the symmetrical arch to be altered, as by the effect of vertical shear.

the arch rib axially, to reduce the horizontal reaction, and also to change the bending moments at the ends.

If the effect of axial stress is considered, the development of the equation for horizontal thrust, in the unsymmetrical arch, is somewhat long, and results in Equation (1).

$$H_1 = \frac{\Sigma_o^{\,l} m_x \Delta \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta}\right)}{\Sigma_o^{\,l} y \Delta \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta}\right)} - \frac{\left(\left[\frac{M_2 - M_1}{l} + R_1\right] \cdot \Sigma_o^{\,l} \frac{\delta x \cdot \delta y}{F_x \cdot \delta s} - P \Sigma_a^{\,l} \frac{\delta x \cdot \delta y}{F_x \cdot \delta s}\right)}{+ \Sigma_o^{\,l} \frac{(\delta x)^2}{F_x \cdot \delta s} + \frac{h}{l} \Sigma_o^{\,l} \frac{\delta x \cdot \delta y}{F_x \cdot \delta s}} \quad \ldots\ldots (1)$$

The horizontal reaction, for either the symmetrical or the unsymmetrical arch, neglecting the effect of axial stress, is given by Equation (2),

$$H = \frac{\Sigma m_x \Delta \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta}\right)}{\Sigma y \Delta \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta}\right)} \quad \ldots\ldots (2)$$

The equation for the horizontal reaction in a symmetrical arch, including the effect of axial stress, is,

$$H_1 = \frac{\Sigma m_x \Delta \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta}\right)}{\Sigma y \Delta \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta}\right) + \Sigma \frac{(\delta x)^2}{F_x \cdot \delta s}} \quad \ldots\ldots (3)$$

In all cases the limits of summation in Equations (2) and (3) are from *0* to *l*, or across the whole span of the arch.

Referring to Equation (1): The expression, $\frac{M_2 - M_1}{l} + R_1$, which occurs in the numerator, is the vertical reaction at the left end of the arch.

The expression, $\Sigma_o^{\,l} \frac{\delta x \cdot \delta y}{P_x \cdot \delta s}$, becomes zero for symmetrical arches, because, for each positive value of δy, there is a corresponding negative value. The ratio, $\frac{\delta y}{\delta s}$, in this expression, is the sine of the angle of slope of the arch segment under consideration. For nearly symmetrical arches, the value of $\frac{\delta y}{\delta s}$ will be very small, and it is relatively small in any case.

The values of M_2 and M_1 cannot easily be obtained directly, as they depend on the value of H_1, but, as the term into which they enter is comparatively small and often negligible, the values obtained for the case in which the axial stress is neglected may be used as a first approximation, to evaluate the expression in which they occur, and may be corrected later, if desired.

The expression, $P \cdot \Sigma_a^l \frac{\delta x \cdot \delta y}{F_x \cdot \delta s}$, in the numerator, represents the deduction from the preceding expression, and is required by the fact that the part of the arch on the left of the loaded point is subject to a different axial stress than that on the right. For the same reasons as previously given for the expression, $\Sigma_o^l \frac{\delta x \cdot \delta y}{F_x \cdot \delta s}$, the expression, $P \Sigma_a^l \frac{\delta x \cdot \delta y}{F_x \cdot \delta s}$, becomes negligible in some cases.

The expression, $\Sigma_o^l \frac{(\delta x)^2}{F_x \cdot \delta s}$, in the denominator, involves the ratio, $\frac{\delta x}{\delta s}$, which is the cosine of the angle of slope of the arch segment under consideration. This expression should always be evaluated when the effect of the axial stress is to be considered.

The expression, $\frac{h}{l} \Sigma_o^l \frac{\delta x \cdot \delta y}{F_x \cdot \delta s}$, in the denominator, is relatively small, and in some cases will be negligible.

If all the terms which have been indicated as relatively small are omitted, the expression for H_1 for unsymmetrical arches reduces to that for symmetrical arches.

In the case of unsymmetrical Arch No. 1-*A* (Fig. 3), the value of H_1, when the effect of axial stress is considered, is 97.8% of the value when the effect of axial stress is neglected, and is 99.3% of the value of H_1 for the corresponding symmetrical arch, when the effect of axial stress is considered.

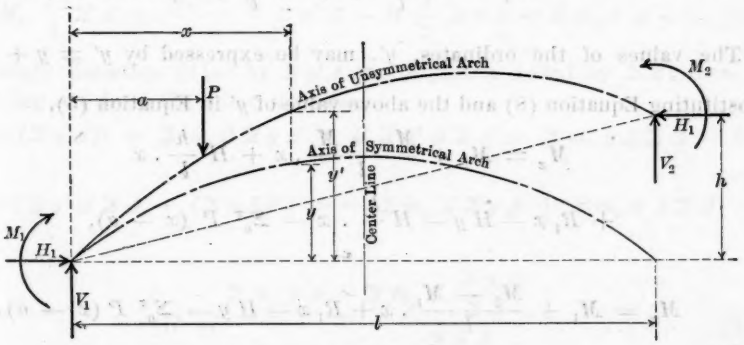


FIG. 3.

In general, it appears that, if the symmetrical arch is of such a shape and character that the effect of axial stress may be neglected, this effect may also be neglected in the unsymmetrical arch obtained from it; and, conversely, if the effect of the axial stress should be considered in one arch, it should also be considered in the other.

PART II

Consider an unsymmetrical arch, the shape of the axis of which is such that it may be obtained from the axis of a symmetrical arch, by raising one end above the other, the vertical ordinates from points on the chord to points

on the arch axis remaining the same. Let the effect of axial stress be neglected.

Let $\Delta \left(= \frac{\delta s}{I}\right)$ have the same values in the unsymmetrical arch as were used in the original arch; let x and y be the values of the co-ordinates of any point in the original arch axis, the origin of the co-ordinates being at the left end of the axis, and let x and y' be the values of the corresponding co-ordinates in the unsymmetrical arch axis.

Assuming vertical loads,

$$H_1 = H_2 = H \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

If M_x represents the bending moment at any section of the arch the abscissa of which is x,

$$M_x = M_1 + V_1 x - H y' - \Sigma_0^x P (x - a) \ldots\ldots\ldots\ldots (5)$$

When $x = l$,

$$M_x = M_2 = M_1 + V_1 l - H h - \Sigma_0^l P (l - a) \ldots\ldots\ldots\ldots (6)$$

From Equation (3),

$$V_1 = \frac{M_2 - M_1}{l} + \frac{H h}{l} + \Sigma_0^l P \frac{l - a}{l} \ldots\ldots\ldots\ldots\ldots (7)$$

Let $\Sigma_0^l P \frac{l - a}{l}$ be called R_1; then,

$$V_1 = \frac{M_2 - M_1}{l} + H \frac{h}{l} + R_1 \ldots\ldots\ldots\ldots\ldots (8)$$

The values of the ordinates, y', may be expressed by $y' = y + \frac{h}{l} \cdot x$.

Substituting Equation (8) and the above value of y' in Equation (5),

$$M_x = M_1 + \frac{M_2 - M_1}{l} \cdot x + H \frac{h}{l} \cdot x$$

$$+ R_1 x - H y - H \frac{h}{l} \cdot x - \Sigma_0^x P (x - a),$$

or,

$$M_x = M_1 + \frac{M_2 - M_1}{l} \cdot x + R_1 x - H y - \Sigma_0^x P (x - a) \ldots\ldots (9)$$

Let,

$$R_1 x - \Sigma_0^x P (x - a) = m_x;$$

then,

$$M_x = M_1 + \frac{M_2 - M_1}{l} \cdot x - H y + m_x \ldots\ldots\ldots\ldots (10)$$

The conditional equations for a fixed arch, with the notation as herein used, are:

$$\Sigma_0^l M_x \Delta = 0;$$

$$\Sigma_0^l M_x y \Delta = 0;$$

and,

$$\Sigma_0^l M_x x \Delta = 0.$$

The first of these equations expresses the condition that the total central angle subtended by the arch does not change under loading; the second expresses the condition that the length of span remains constant under loading; and the third expresses the condition that the abutments do not yield vertically under loading.

Substituting from Equation (10) in these three equations of condition, we have (the summations being understood to be from 0 to l in all cases, unless otherwise stated):

$$M_1 \Sigma \Delta + \frac{M_2 - M_1}{l} \Sigma x \Delta - H \Sigma y \Delta + \Sigma m_x \Delta = 0 \ldots\ldots (11)$$

$$M_1 \Sigma y \Delta + \frac{M_2 - M_1}{l} \Sigma x y \Delta - H \Sigma y^2 \Delta + \Sigma m_x y \Delta = 0 \ldots (12)$$

$$M_1 \Sigma x \Delta + \frac{M_2 - M_1}{l} \Sigma x^2 \Delta - H \Sigma x y \Delta + \Sigma m_x x \Delta = 0 \ldots\ldots (13)$$

As shown in Howe's "Symmetrical Masonry Arches", $\Sigma x \Delta = \frac{1}{2} l \Sigma \Delta$, and $\Sigma x y \Delta = \frac{1}{2} l \Sigma y \Delta$. Substituting these values in Equations (11), (12), and (13), we have,

$$M_1 \Sigma \Delta + \frac{M_2 - M_1}{2} \Sigma \Delta - H \Sigma y \Delta + \Sigma m_x \Delta = 0 \ldots\ldots (11a)$$

$$M_1 \Sigma y \Delta + \frac{M_2 - M_1}{2} \Sigma y \Delta - H \Sigma y^2 \Delta + \Sigma m_x y \Delta = 0 \ldots (12a)$$

$$M_1 \frac{l}{2} \Sigma \Delta + \frac{M_2 - M_1}{l} \Sigma x^2 \Delta - H \frac{l}{2} \Sigma y \Delta + \Sigma m_x x \Delta = 0 \ldots (13a)$$

Multiply Equation (11a) by $\Sigma y \Delta$ and Equation (12a) by $\Sigma \Delta$; then, by subtracting one from the other, we have,

$$- H (\Sigma y \Delta)^2 + \Sigma m_x \Delta \Sigma y \Delta + H \Sigma y^2 \Delta \Sigma \Delta - \Sigma m_x y \Delta \Sigma \Delta = 0$$

or,

$$H [\Sigma y^2 \Delta \Sigma \Delta - (\Sigma y \Delta)^2] = - \Sigma m_x \Delta \Sigma y \Delta + \Sigma m_x y \Delta \Sigma \Delta$$

or,

$$H = \frac{\Sigma m_x y \Delta - \Sigma m_x \Delta \dfrac{\Sigma y \Delta}{\Sigma \Delta}}{\Sigma y^2 \Delta - \Sigma y \Delta \dfrac{\Sigma y \Delta}{\Sigma \Delta}} \ldots\ldots\ldots\ldots\ldots (14a)$$

or,

$$H = \frac{\Sigma m_x \Delta \left[y - \dfrac{\Sigma y \Delta}{\Sigma \Delta} \right]}{\Sigma y \Delta \left[y - \dfrac{\Sigma y \Delta}{\Sigma \Delta} \right]} \ldots\ldots\ldots\ldots\ldots (14b)$$

Equation (14b) is the same as Equation (60) in "Symmetrical Masonry Arches", which indicates that the horizontal thrust for the unsymmetrical arch will be the same as that for the symmetrical arch from which it was obtained, when the loadings are identical.

Multiplying Equation (11*a*) by $\Sigma\, x^2\, \Delta$ and Equation (13*a*) by $\frac{l}{2}\, \Sigma\, \Delta$, they become:

$$M_1\, \Sigma\, \Delta\, \Sigma\, x^2\, \Delta + \frac{M_2 - M_1}{2}\, \Sigma\, \Delta\, \Sigma\, x^2\, \Delta - H\, \Sigma\, y\, \Delta\, \Sigma\, x^2\, \Delta + \Sigma\, m_x\, \Delta\, \Sigma\, x^2\, \Delta = 0 \quad \ldots\ldots\ldots\ldots (15)$$

$$\frac{l}{4}\, M_1\, l^2\, \Sigma\, \Delta\, \Sigma\, \Delta + \frac{M_2 - M_1}{2}\, \Sigma\, x^2\, \Delta\, \Sigma\, \Delta - H\, \frac{l^2}{4}\, \Sigma\, y\, \Delta\, \Sigma\, \Delta + \frac{l}{2}\, \Sigma\, m_x\, x\, \Delta\, \Sigma\, \Delta = 0 \quad \ldots\ldots\ldots\ldots (16)$$

Subtracting Equation (16) from Equation (15), we have,

$$M_1 \left[\Sigma\, \Delta\, \Sigma\, x^2\, \Delta - \frac{l^2}{4}\, \Sigma\, \Delta\, \Sigma\, \Delta\right] + H \left[\frac{l^2}{4}\, \Sigma\, y\, \Delta\, \Sigma\, \Delta - \Sigma\, y\, \Delta\, \Sigma\, x^2\, \Delta\right] + \Sigma\, m_x\, \Delta\, \Sigma\, x^2\, \Delta - \frac{l}{2}\, \Sigma\, m_x\, x\, \Delta\, \Sigma\, \Delta = 0$$

or,

$$M_1 = H\, \frac{\Sigma\, y\, \Delta \left[\frac{l^2}{4}\, \Sigma\, \Delta - \Sigma\, x^2\, \Delta\right]}{\Sigma\, \Delta \left[\frac{l^2}{4}\, \Sigma\, \Delta - \Sigma\, x^2\, \Delta\right]} - \frac{\Sigma\, m_x\, \Delta \left[\frac{l}{2}\, x\, \Sigma\, \Delta - \Sigma\, x^2\, \Delta\right]}{\Sigma\, \Delta \left[\frac{l^2}{4}\, \Sigma\, \Delta - \Sigma\, x^2\, \Delta\right]}$$

or,

$$M_1 = H\, \frac{\Sigma\, y\, \Delta}{\Sigma\, \Delta} - \frac{\Sigma\, m_x\, \Delta \left[\frac{l}{2}\, x - \frac{\Sigma\, x^2\, \Delta}{\Sigma\, \Delta}\right]}{\Sigma\, \Delta \left[\frac{l^2}{4} - \frac{\Sigma\, x^2\, \Delta}{\Sigma\, \Delta}\right]}$$

or,

$$M_1 = H\, \frac{\Sigma\, y\, \Delta}{\Sigma\, \Delta} - \frac{\Sigma\, m_x\, \Delta}{\Sigma\, \Delta} - \frac{\Sigma\, m_x\, \Delta \left(x - \frac{l}{2}\right)}{\Sigma\, \Delta \left(\frac{l^2}{4} - \frac{\Sigma\, x^2\, \Delta}{\Sigma\, \Delta}\right)} \cdot \frac{l}{2} \quad \ldots\ldots (17)$$

By comparing Equation (17) with Equation (68) in "Symmetrical Masonry Arches", it is seen that the moment at the support, in the case of the unsymmetrical arch, is the same as in the case of the symmetrical arch from which it was obtained, it being understood that the loadings are the same in the two cases.

PERIODIC FLUCTUATIONS OF RAINFALL IN HAWAII

By Joel B. Cox,* Assoc. M. Am. Soc. C. E.

Synopsis

In Hawaii, where the conditions are especially favorable for a close relationship between insolation and rainfall, there is a periodic fluctuation of rainfall of great importance and considerable regularity. Periods of 1 year, 3.7 years, 11.1 years, and 33 years have been noted. Quite a close correlation can be found between this rainfall and the annual change in pyrheliometric measurements and sun-spot numbers. Forecasts obtained by the use of these data may be of importance to the sugar plantations.

General

For many years, astronomers and meteorologists have been aware that the sun is subject to periodic fluctuations, of considerable regularity, affecting many of its observational characteristics—notably sun-spots and solar prominences—and that in many cases an important correlation could be demonstrated between such fluctuations of solar phenomena and terrestrial conditions, magnetic, electrical, and meteorological. Striking demonstration of this correlation and of its cyclic character is given in Bigelow's "Atmospheric Circulation and Radiation".

It is probable that many have suspected a relationship between solar cycles and terrestrial rainfall, but any close correlation of engineering value appeared to be improbable in the extra-tropical zones, where precipitation is determined so largely by the accidental coincidences of cyclonic disturbance and topographic conditions, and especially on great continents, where a considerable part of the precipitation is derived from evaporation from land areas.

On the Hawaiian Islands, the conditions are different, and are extremely favorable for a sensitive response to solar changes. During most of the year, cyclonic disturbances are unknown, and, over thousands of miles of surrounding ocean, the weather conditions are almost uniform. The rainfall of the islands occurs on the northeast or windward slopes of high mountains which rise 4 000 to 13 000 ft. above the sea, intercepting the constant sweep of the northeast trade winds and precipitating most of their moisture. Annual rainfalls ranging from 100 in. at the shore to 500 in. at an elevation of about 5 000 ft. are common, and these heavy rainfalls are distributed very uniformly throughout the year. Thus, the precipitation, primarily, is due to the evapora-

Note.—Written discussion on this paper, which will not be presented at any meeting of the Society, will be closed with the **January, 1924,** *Proceedings.* When finally closed, the paper, with discussion, will be published in *Transactions.*

* Engr. with East Maui Irrig. Co., Paia, Maui, Hawaii.

tion from thousands of miles of tropical ocean, and is caused directly by the almost constant movement of the northeast trade winds.

The practical importance of a knowledge of the causes of fluctuations of rainfall is great, especially if such fluctuations can thereby be predicted to any extent. The major industry of the islands is the production of cane sugar, the growth of the cane being under intensive irrigation that requires about 1.0 acre-ft. per month during the crop period of 18 months. The value of the raw sugar produced per acre per crop is about $500, or $80 per 1 000 000 gal. of water. The loss during a dry season is sometimes enormous, especially if a series of wet years has encouraged overplanting.

The writer believes that the statistical study of the rainfall of the Hawaiian Islands herewith presented shows a most important cyclic fluctuation, due to its close relationship to solar changes, and that the regularity of this cyclic change enables predictions of rainfall and water supply to be made with a smaller probable error than by any method hitherto used.

Data

The data used in the following study are the precipitation records on file with the U. S. Weather Bureau, in Honolulu, Hawaii, and with private interests on the Island of Maui, and tables of solar phenomena furnished by the U. S. Weather Bureau, Washington, D. C. Satisfactory rainfall records are generally obtainable only for the 16 years since the establishment of the Bureau in Honolulu in 1905, but a composite record, representing with fair accuracy the average observed rainfall for the islands, has been prepared from the records of ten representative stations having long-term records, and extended to give a 44-year record. These records are given in Tables 1 and 2.

TABLE 1.—Rainfall, etc., at Stations used in Computing 10-Station Composite Record.

Station.	Location.	Elevation.	Length of record, in years.	Mean annual rainfall, in inches.
Hilo	Hawaiian Islands	E. 40	33	139.43
Niulii	" "	NE. 85	35	58.72
Puuomalei	Maui "	N. 1 300	26	79.26
Haleakala Ranch	" "	N. 2 000	29	49.61
Lupe	" "	N. 1 300	24	163.24
Honolulu	Oahu "	SE. 111	44	31.18
Waimanalo	" "	NE. 25	26	45.43
Grove Farm	Kauai "	E. 200	36	47.91
Kilauea	" "	N. 342	35	70.48
Koloa	" "	E. 241	33	64.46
Average	.....	.....	.....	74.97

The precipitation of the islands is almost altogether determined by topography, and records may be grouped in three main types:

1.—Windward.—Occurring on the north and east slopes of the mountains, and due to the trade winds; very heavy on steep slopes with favorable exposure.

TABLE 2.—10-STATION AVERAGE. HAWAIIAN RAINFALL.

	1877.	1878.	1879.	1880.	1881.	1882.	1883.	1884.	1885.
Jan.	.....	0.41	15.00	7.40	3.60	17.00	3.32	4.71	4.21
Feb.	5.06	0.77	4.81	16.50	15.71	10.30	11.45	1.32	5.17
Mar.	1.61	0.31	1.01	7.41	4.45	10.10	1.82	6.87	5.30
Apr.	4.61	3.46	6.00	6.85	3.47	3.19	5.57	12.31	10.40
May	12.48	0.55	5.22	3.50	2.43	3.46	4.28	7.97	15.23
June	0.57	4.95	1.58	4.13	4.80	2.97	2.84	3.78	7.09
July	2.04	1.51	4.13	10.70	4.78	3.35	4.27	7.01	13.07
Aug	1.27	3.93	4.28	4.29	4.92	4.30	8.53	7.84	6.33
Sept	2.69	1.49	1.56	6.88	4.42	4.21	2.49	5.04	7.56
Oct.	1.83	4.27	2.23	7.11	2.29	4.74	12.64	7.25	7.29
Nov.	4.16	2.57	8.46	5.25	14.62	8.69	4.98	2.83	7.77
Dec.	5.63	4.30	21.25	1.40	14.95	8.46	8.91	8.53	7.65
Year	46.22	28.53	75.51	81.42	79.94	80.77	71.10	75.45	97.08

	1886.	1887.	1888.	1889.	1890.	1891.	1892.	1893.	1894.
Jan.	1.69	11.93	4.46	1.77	17.27	6.78	15.18	5.76	6.52
Feb.	2.92	8.63	11.22	1.53	10.42	7.90	5.93	13.32	14.83
Mar.	10.95	5.73	3.80	1.88	25.47	4.17	3.99	9.33	8.02
Apr.	5.86	5.07	5.21	4.04	9.94	6.36	5.16	6.34	5.84
May	7.66	9.29	5.10	5.58	9.71	2.76	5.77	4.29	1.29
June	4.69	2.06	4.22	3.83	4.08	3.54	5.06	3.54	2.23
July	4.48	4.54	4.42	5.75	2.91	4.61	3.99	2.92	3.65
Aug	3.81	5.14	5.37	6.01	4.79	5.19	4.37	2.48	1.83
Sept.	11.12	2.35	6.37	6.70	3.58	6.56	2.72	2.64	2.04
Oct.	3.21	5.35	8.10	4.83	3.55	8.35	6.09	4.32	2.87
Nov.	13.60	13.28	7.43	5.90	8.67	4.38	5.66	6.96	16.39
Dec	5.14	19.80	3.37	7.09	6.51	6.84	7.59	5.04	7.25
Year	75.13	87.18	69.06	54.89	106.90	67.43	71.51	66.94	72.76

	1895.	1896.	1897.	1898.	1899.	1900.	1901.	1902.	1903.
Jan.	5.00	3.94	2.68	10.75	1.12	2.46	5.78	2.00	6.86
Feb.	3.23	5.42	4.85	6.48	4.14	4.28	12.82	4.25	5.83
Mar.	4.78	6.51	4.53	17.97	9.37	2.82	10.84	32.24	3.25
Apr.	5.74	6.24	2.05	3.36	5.60	6.37	4.81	6.43	9.07
May	3.12	5.29	2.91	4.36	7.50	6.02	4.07	7.08	2.46
June	2.71	3.94	2.00	3.33	3.08	2.31	2.63	4.14	3.29
July	2.77	2.92	3.37	4.80	2.65	5.15	4.67	4.05	7.56
Aug	9.13	4.48	4.80	3.10	3.33	5.21	2.69	7.10	4.26
Sept.	6.88	2.77	3.29	3.43	1.77	2.94	2.10	4.28	7.09
Oct.	3.84	4.57	4.18	3.23	9.42	8.82	5.60	6.63	6.02
Nov.	7.29	6.69	6.07	2.68	3.07	12.13	11.35	10.85	6.32
Dec.	7.99	8.25	4.11	5.42	2.43	3.41	9.50	15.27	5.17
Year	62.49	61.02	44.84	68.10	53.47	61.91	76.85	104.32	67.18

	1904.	1905.	1906.	1907.	1908.	1909.	1910.	1911.	1912.
Jan.	11.05	2.94	4.99	13.00	2.79	4.35	9.05	9.92	1.63
Feb.	18.81	2.08	1.30	7.58	8.10	6.19	3.24	13.07	6.76
Mar.	9.08	4.33	5.10	9.04	9.49	15.82	6.27	6.68	6.24
Apr	11.59	6.04	4.99	4.71	5.04	5.39	5.79	5.42	7.86
May	2.63	5.12	4.07	4.66	3.62	4.49	5.59	6.16	3.14
June	2.96	3.94	2.46	4.21	3.25	3.82	5.87	5.80	2.57
July	4.98	3.89	4.88	5.83	3.91	6.80	5.10	3.67	3.54
Aug	6.30	6.98	8.70	13.81	5.26	2.45	6.98	4.80	2.93
Sept.	3.49	8.51	3.81	6.77	5.60	3.49	4.68	10.40	2.59
Oct.	3.49	4.62	4.49	5.87	4.17	4.52	4.14	2.95	6.31
Nov.	5.96	8.78	6.78	6.14	3.56	2.51	7.70	5.33	6.18
Dec.	5.54	5.42	16.31	3.94	6.81	10.25	11.19	5.66	8.09
Year	85.89	62.60	67.88	85.56	61.59	70.07	75.61	79.86	57.87

TABLE 2.—(*Continued.*)

	1913.	1914.	1915.	1916.	1917.	1918.	1919.	1920.	1921.
Jan.	6.40	7.53	2.18	18.26	7.90	10.66	3.25	6.76	23.51
Feb.	4.30	1.66	6.14	2.95	4.47	11.93	2.66	1.64	2.33
Mar.	3.96	8.02	2.39	8.57	12.89	14.59	5.28	11.44	5.70
Apr.	7.05	8.06	10.57	6.19	6.40	20.09	3.64	4.28	4.68
May.	4.85	12.62	2.34	13.56	5.92	6.18	3.74	1.86	1.76
June.	6.08	8.36	4.77	6.33	3.20	4.95	2.15	2.48	2.05
July.	3.03	9.47	4.34	5.17	2.99	8.79	4.28	3.77	4.46
Aug.	4.53	11.64	2.71	5.76	3.06	7.67	4.58	4.77	3.94
Sept.	2.95	14.86	7.15	4.76	2.84	2.47	4.20	4.46	3.30
Oct.	3.53	5.73	8.23	4.40	3.73	3.84	5.20	5.85	5.85
Nov.	13.26	9.10	14.01	7.04	6.21	9.71	3.48	6.10	11.90
Dec.	6.33	10.23	11.44	14.61	5.03	11.76	6.58	10.26	14.31
Year.	66.27	107.28	76.27	97.60	64.62	112.64	49.04	63.68	83.79

2.—Leeward.—Occurring on the southwest shores of the lower islands (less than 10 000 ft.); no rain from trade winds, and arid in type, with long dry periods, and occasional very heavy deluges at times of southern storms, or "Konas" which occur in winter.

3.—Monsoon.—Occurring on the southwest slopes of high mountains, 10 000 ft. and more, where the barrier against the trades is so high that the normal wind is from the southwest; characterized by steady summer rains.

Plate VII shows the distribution of these three types of climate, and the isohyetose lines for 1920.

Method of Extending Records

As the method of extension of the records used in this study is new, and may prove of interest and value to others, it is given in some detail.

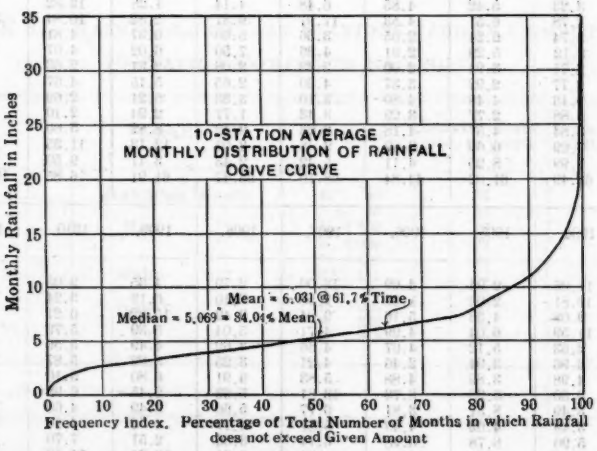
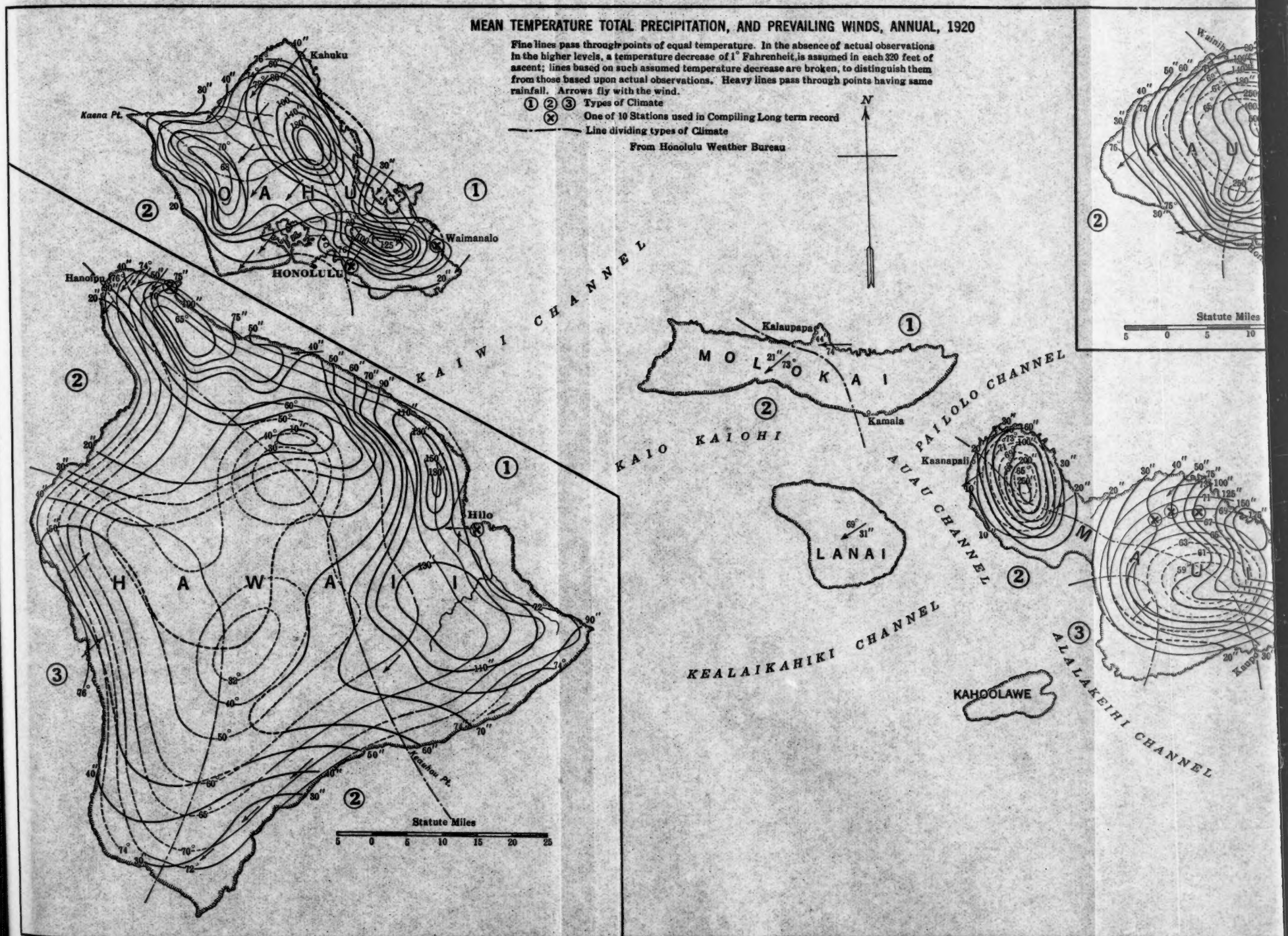


FIG. 1.

It was apparent that the distribution of the monthly rainfall at the various stations followed skew frequency curves of the general type shown in Figs. 1 and 2. To extend short-period records by comparison with the 44-year


MEAN TEMPERATURE TOTAL PRECIPITATION, AND PREVAILING WINDS, ANNUAL, 1920
Fine lines pass through points of equal temperature. In the absence of actual observations in the higher levels, a temperature decrease of 1° Fahrenheit, is assumed in each 320 feet of ascent; lines based on such assumed temperature decrease are broken, to distinguish them from those based upon actual observations. Heavy lines pass through points having same rainfall. Arrows fly with the wind.
① ② ③ Types of Climate
One of 10 Stations used in Compiling Long term record
Line dividing types of Climate
From Honolulu Weather Bureau
N
Kahuku
Kaena Pt.
O A H U
Waimanalo
HONOLULU
Hanolpu
KAIWI CHANNEL
Kalaupapa
M O L O K A I
Kamala
KAIO
KAIOHI
PAILOLO CHANNEL
AUAU CHANNEL
Kaanapali
LANAI
H A W A I I
Hilo
Kealakeku Pt.
KEALAIKAHIKI CHANNEL
KAHOOLAWE
ALALAKEIHI CHANNEL
Statute Miles
5 0 5 10 15 20 25
Statute Miles
5 0 5 10

E TOTAL PRECIPITATION, AND PREVAILING WINDS, ANNUAL, 1920

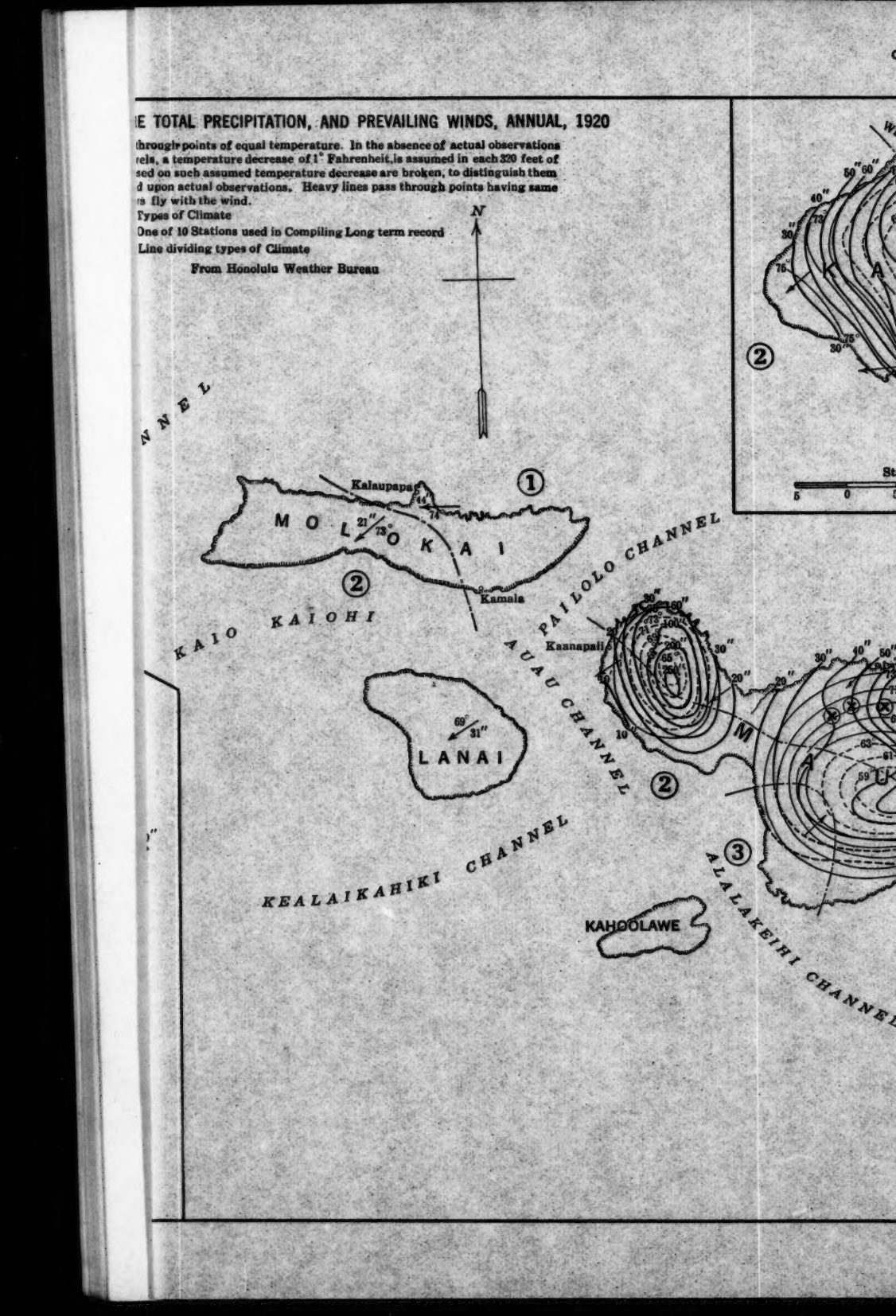
through points of equal temperature. In the absence of actual observations
rels, a temperature decrease of 1° Fahrenheit is assumed in each 320 feet of
sed on such assumed temperature decrease are broken, to distinguish them
d upon actual observations. Heavy lines pass through points having same
rs fly with the wind.

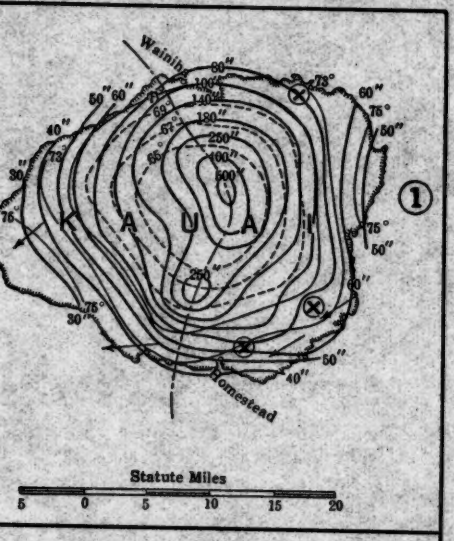
Types of Climate

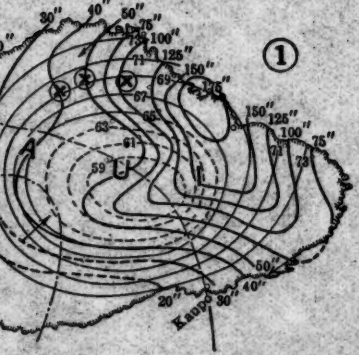
One of 10 Stations used in Compiling Long term record

Line dividing types of Climate

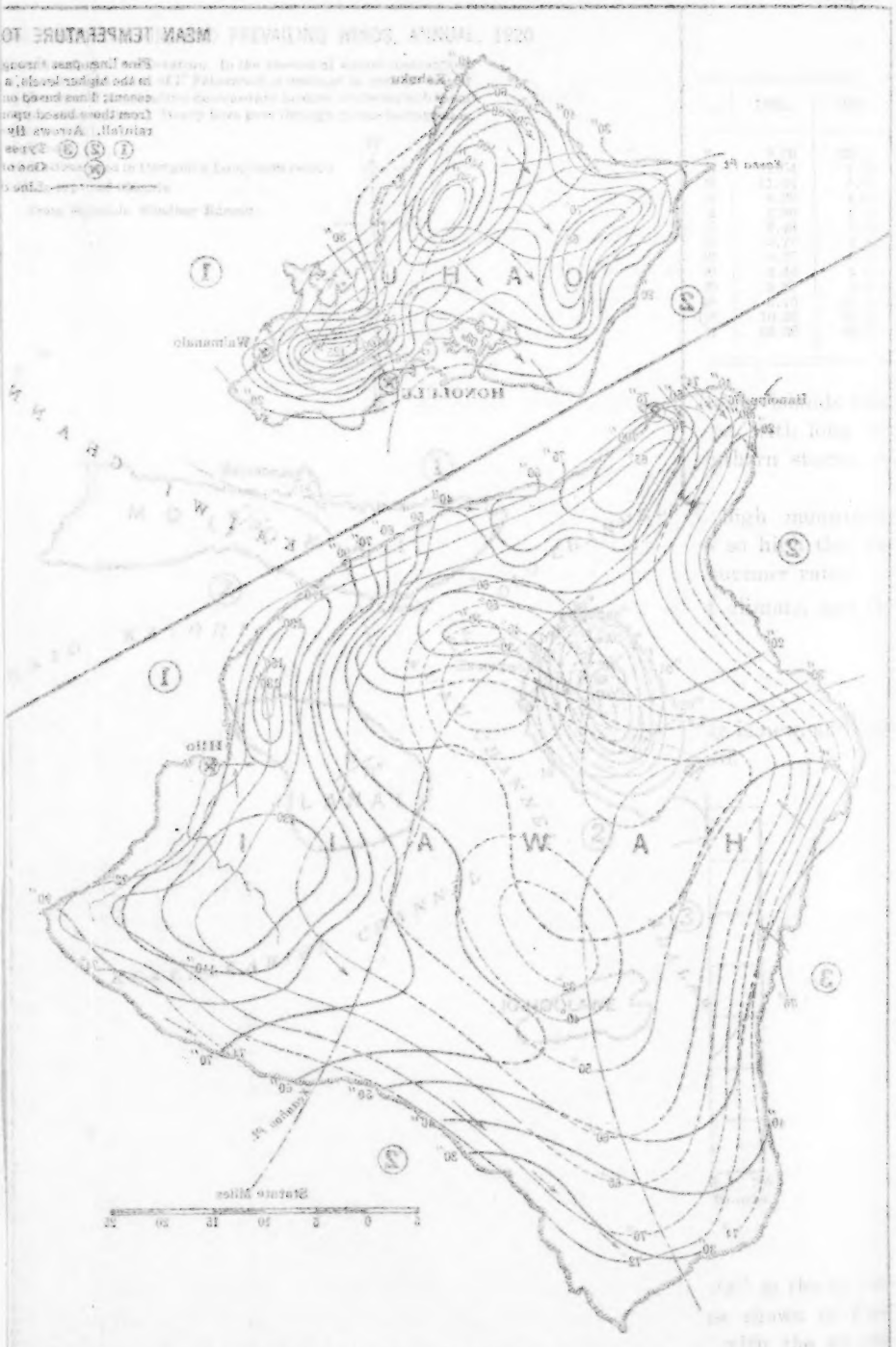
From Honolulu Weather Bureau







I CHANNEL


O A H U
M A U I
H A W A I I
HONOLULU
Hilo
Statute Miles

record (Table 2, Figs. 1, 2, and 10), the following method was used. It is illustrated by a single example, and seemed to give the best results.

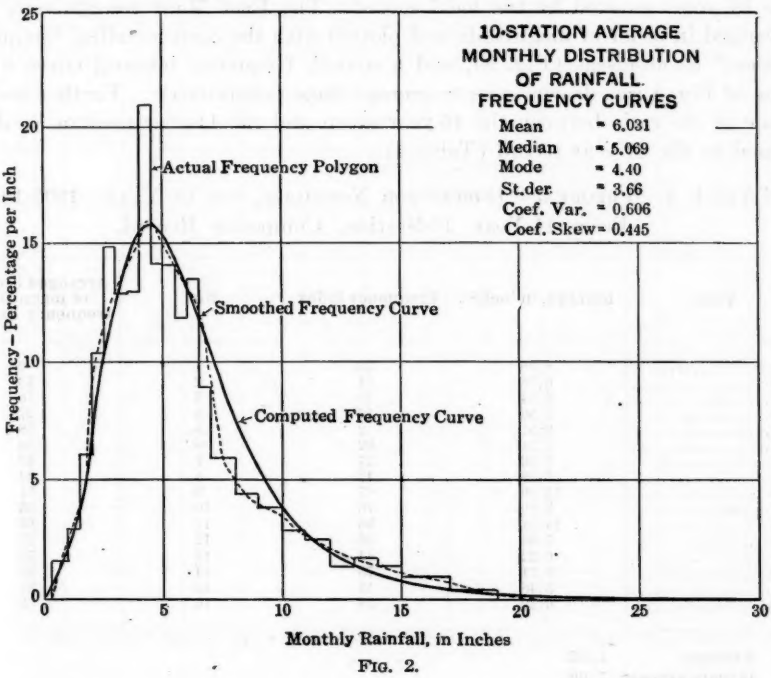


FIG. 2.

TABLE 3.—REPRESENTATIVE HAWAIIAN RAINFALL RECORDS.

		10-station composite.	East Maui, 1 200 ft.	Honolulu.	Hilo.
Length of record, in years		44	16	44	30
Monthly distribution.	Coefficient of variation	0.615	0.611	.....	.....
	Approximate coefficient of skew	1.60	1.34	.....	.....
	True coefficient of skew	0.445	.....	.....	.....
	Mean	6.03	16.87	.....	.....
	Mode	4.40	.....	.....	.....
	Median	5.07	14.92	.....	.....
Annual distribution.	Coefficient of variation	0.239	0.264	0.348	0.187
	Approximate coefficient of skew	0.257	.....	0.370	0.309
	True coefficient of skew	0.419	.....	.....	.....
	Mean	72.56	202.47	28.96	136.80
	Mode	65.3	.....	.....	.....
	Median	70.59	188.43	27.80	133.00

The example presented is from the extension of the records for the rainfall at the 1 200-ft. level on the East Maui water-shed, and is for November. The records were complete for 16 years. Table 4 was first prepared from the 44-year record, giving the "Frequency Index" for November for the 16 years covered by the East Maui record. The "frequency index" is the percentage of the total number of records for November which are less than the given amount. In statistical language, it is the "grade" of Galton's ogive

curve. Thus, of the 44 records for November, the lowest, 6th, 7th, 12th, 17th, 18th, 19th, 20th, 23d, 25th, 28th, 33d, 34th, 35th, 39th, and 42d occurred during the 16 years covered by the local record. The East Maui records were then arranged in order of magnitude and plotted with the corresponding "frequency indices" as abscissas (Fig. 5), and a smooth frequency integral curve of the type of Fig. 1 was drawn so as to average these points fairly. Further use was made of the ratio between the 16-year mean and the 44-year median, as determined by the 44-year record (Table 4).

TABLE 4.—Frequency Indices for November, for 16 Years, 1905-1920. From 44-Year, 10-Station, Composite Record.

Year.	Rainfall, in inches.	Frequency index.	No.	Arranged in order of magnitude frequency indices.
1905	8.78	73	1	2
1906	6.78	51	2	13
1907	6.14	40	3	16
1908	3 56	16	4	27
1909	2.51	2	5	38
1910	7.71	62	6	40
1911	5.33	27	7	42
1912	6.18	42	8	44
1913	13.26	87	9	51
1914	9.10	76	10	56
1915	14.01	93	11	62
1916	7.94	56	12	73
1917	6.22	44	13	76
1918	9.71	78	14	78
1919	3.48	13	15	87
1920	6.10	38	16	93

Average	7.245
44 years average	7.498
44 years median	6.736
Ratio $\frac{\text{44 years average}}{\text{16 years average}}$	1.0349
Ratio $\frac{\text{44 years median}}{\text{16 years average}}$	0.9297

If the probable East Maui rainfall for November, 1901, is desired, for which the frequency index of the 44-year record was 82, it is read from Fig. 5 as 26.5 in.

The advantage of this method of extension is that records having differing coefficients of variation may be compared without the extremely high occasional months of the record—with a high coefficient of variation—influencing a short-term record unduly, as would be the case if averages were used. It is believed that no other method so well allows the comparison of records differing widely in their coefficients of variation, but influenced by similar general conditions affecting rainfall.

Analysis of Periodic Fluctuations.—The general method of analysis was to investigate and determine every periodic variation recognizable, and then eliminate its effects before searching for any other periodic effect. Thus, the annual cycle had first to be determined and allowance made therefor. This was done by averaging the records of a station for a given month and determining the monthly mean and frequency distribution. The effect of the annual cycle could then be eliminated from the record. This transformed record was next

analyzed for the effect of the 3.7-year cycle, which had already been suspected. The removal of the effect of the 3.7-year cycle, in turn, gave data on the existence and importance of possible periodic fluctuations of longer terms. Finally, an investigation of the practical result of the application of the periodic factors to a specific record was made.

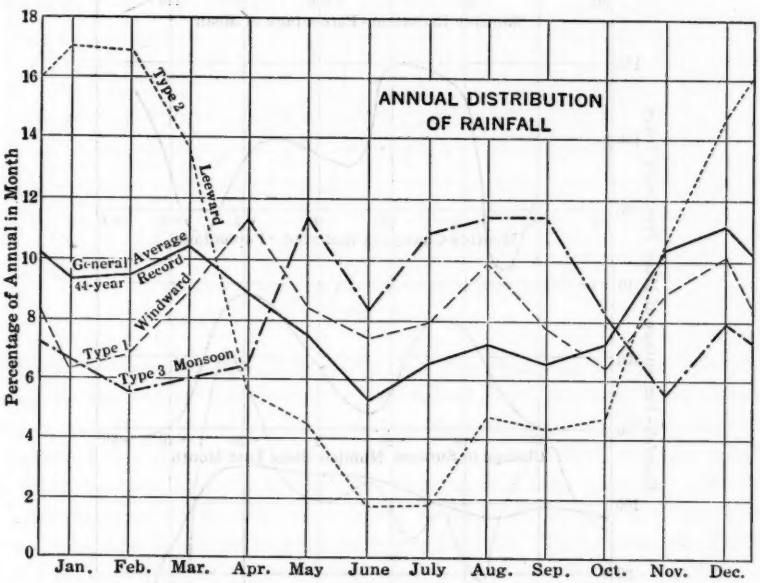


FIG. 3.

Annual Cycle.—The annual distribution of rainfall for records representing the three types of climate referred to previously is shown in Fig. 3. Having given such an annual curve for any station, its record for a given month, originally stated in inches of rainfall, may be reduced to:

(*a*).—Its departure, in inches, from the mean for the given month. This method is used in the Weather Bureau publications.

(*b*).—Its percentage of the mean for the given month.

(*c*).—Its frequency index, as previously defined and as determined by the frequency distribution for the given month.

Each of these three forms removes the effect of the annual cycle, and each was used in at least one of the curves of Fig. 6.

3.7-Year Cycle.—This cycle is so prominent in Hawaiian records that its accidental discovery was the immediate cause of the study preceding this paper, and the preliminary compilation of records and comparison with Bigelow's curves,* brought the adoption of 3.7 years (or 44.4 months) as its average period.

As with the annual cycle, the effect was determined by tabulation. January 1, 1900, was adopted as datum, and Table 5 was prepared, giving to each month a number corresponding to its position in the assumed 3.7-year cycle.

* "Atmospheric Circulation and Radiation", Bigelow, p. 338.

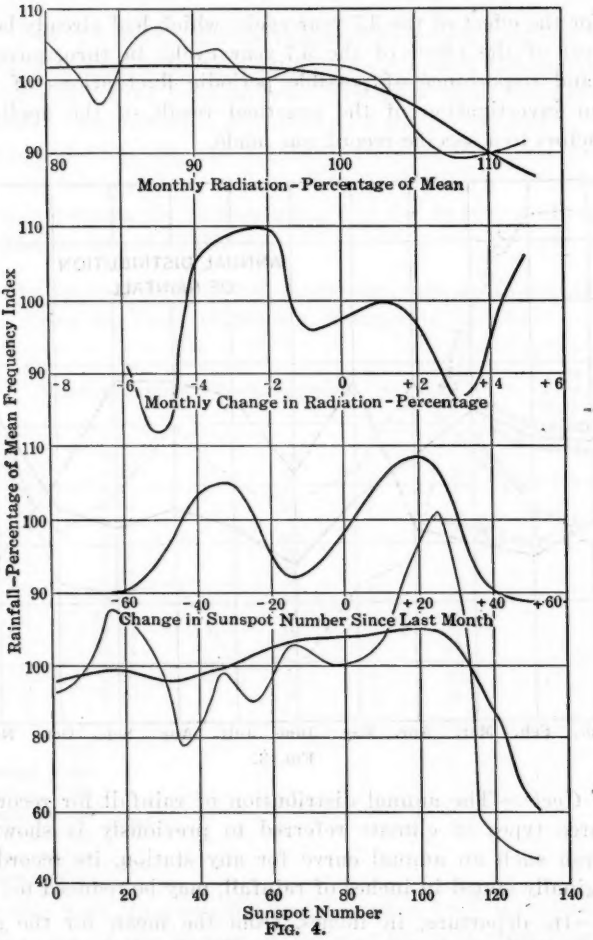

Monthly Radiation - Percentage of Mean
Monthly Change in Radiation - Percentage
Change in Sunspot Number Since Last Month
Rainfall - Percentage of Mean Frequency Index
Sunspot Number

FIG. 4.

The records were treated, by one of the three methods already outlined, to remove the effect of the annual cycle, and smoothed to minimize accidental effects.

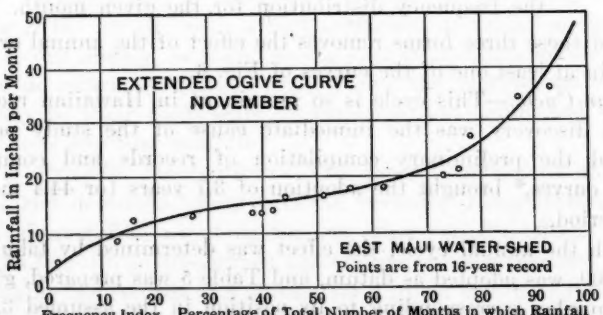

EXTENDED OGIVE CURVE
NOVEMBER
EAST MAUI WATER-SHED
Points are from 16-year record
Rainfall in Inches per Month
Frequency Index. Percentage of Total Number of Months in which Rainfall does not exceed Given Amount

FIG. 5.

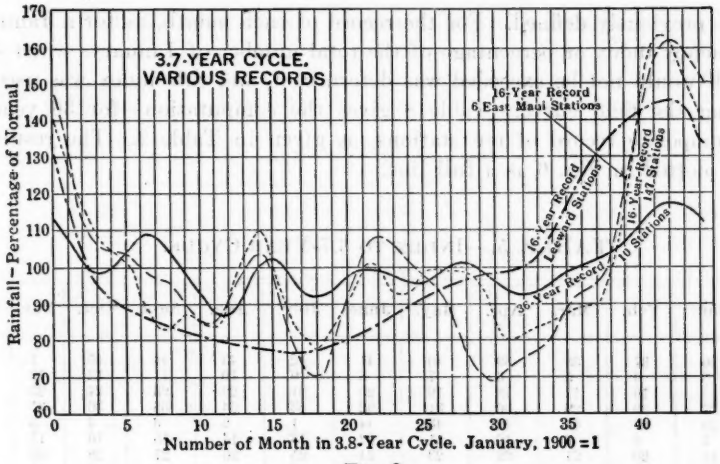


FIG. 6.

Method 1.—Departures.—The curve for the Weather Bureau record was prepared by the use of the departure, in inches, from the long-term mean for the given month, as given in the Weather Bureau reports. These departures were collected for all months having the same cycle number, and the resulting curve was smoothed to include adjacent months and thus gain the advantages of a larger number of records for a given section of the curve. This method was used with the Weather Bureau average, Table 6. This curve, on Fig. 6, is marked "16-Year Record-147 Stations", and is indicated by a dotted line.

The first column in Table 6 is the month number in the 3.7-year cycle, as taken from Table 5. The second column is the average departure, in inches, for all the months on record having the same month number. The next six columns show the method used for smoothing. Column 3 is the sum of the two adjacent figures in Column 2. Column 4 is the sum of the two adjacent figures in Column 3. Column 5 is Column 4 divided by 4. Column 6 is the sum of the two adjacent figures in Column 5. Column 7 is the sum of the two adjacent figures in Column 6. Column 8 is Column 7 divided by 4. This gives the smoothed values of Column 2 by the smoothing formula:

$$c' = \frac{a + 4b + 6c + 4d + e}{16}$$

Column 9 is the equivalent percentage of the monthly mean for plotting in Fig. 6.

Method 2.—Percentage of Averages.—The rainfall for each month of a record was determined as a percentage of the mean for this month, the monthly mean having been established by as long a period as possible. These percentages were then collected and treated as in Method 1. Table 7 shows the computation for the East Maui rainfall, plotted as a line of dashes in Fig. 6.

Method 3.—Frequency Indices.—As both the preceding methods allowed accidental variations, such as isolated months of very high rainfall, unduly to influence the resulting curves, additional work was done by using the frequency

index, as previously defined. For the record of each month, as for a January, the frequency index or percentage of the total number of Januarys when such a rainfall would not be exceeded was determined, and this index was used in computing the final curve. Table 8 gives the computation—for 36 years—of the composite record of ten stations, as given in Table 2. The resulting curve is plotted in Fig. 6 as a full line.

TABLE 5.—INDEX TO 3.7-YEAR CYCLE.

Year.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1877	36	37	38	39	40	41	42	43	44	45	1	2
1878	3	4	5	6	7	8	9	10	11	12	13	14
1879	15	16	17	18	19	20	21	22	23	24	25	26
1880	27	28	29	30	31	32	33	34	35	36	37	38
1881	39	40	41	42	43	44	1	2	3	4	5	6
1882	7	8	9	10	11	12	13	14	15	16	17	18
1883	19	20	21	22	23	24	25	26	27	28	29	30
1884	31	32	33	34	35	36	37	38	39	40	41	42
1885	43	44	1	2	3	4	5	6	7	8	9	10
1886	11	12	13	14	15	16	17	18	19	20	21	22
1887	23	24	25	26	27	28	29	30	31	32	33	34
1888	35	36	37	38	39	40	41	42	43	44	45	1
1889	2	3	4	5	6	7	8	9	10	11	12	13
1890	14	15	16	17	18	19	20	21	22	23	24	25
1891	26	27	28	29	30	31	32	33	34	35	36	37
1892	38	39	40	41	42	43	44	1	2	3	4	5
1893	6	7	8	9	10	11	12	13	14	15	16	17
1894	18	19	20	21	22	23	24	25	26	27	28	29
1895	30	31	32	33	34	35	36	37	38	39	40	41
1896	42	43	44	45	1	2	3	4	5	6	7	8
1897	9	10	11	12	13	14	15	16	17	18	19	20
1898	21	22	23	24	25	26	27	28	29	30	31	32
1899	33	34	35	36	37	38	39	40	41	42	43	44
1900	1	2	3	4	5	6	7	8	9	10	11	12
1901	13	14	15	16	17	18	19	20	21	22	23	24
1902	25	26	27	28	29	30	31	32	33	34	35	36
1903	37	38	39	40	41	42	43	44	1	2	3	4
1904	5	6	7	8	9	10	11	12	13	14	15	16
1905	17	18	19	20	21	22	23	24	25	26	27	28
1906	29	30	31	32	33	34	35	36	37	38	39	40
1907	41	42	43	44	45	1	2	3	4	5	6	7
1908	8	9	10	11	12	13	14	15	16	17	18	19
1909	20	21	22	23	24	25	26	27	28	29	30	31
1910	32	33	34	35	36	37	38	39	40	41	42	43
1911	44	1	2	3	4	5	6	7	8	9	10	11
1912	12	13	14	15	16	17	18	19	20	21	22	23
1913	24	25	26	27	28	29	30	31	32	33	34	35
1914	36	37	38	39	40	41	42	43	44	45	1	2
1915	3	4	5	6	7	8	9	10	11	12	13	14
1916	15	16	17	18	19	20	21	22	23	24	25	26
1917	27	28	29	30	31	32	33	34	35	36	37	38
1918	39	40	41	42	43	44	1	2	3	4	5	6
1919	7	8	9	10	11	12	13	14	15	16	17	18
1920	19	20	21	22	23	24	25	26	27	28	29	30
1921	31	32	33	34	35	36	37	38	39	40	41	42
1922	43	44	1	2	3	4	5	6	7	8	9	10
1923	11	12	13	14	15	16	17	18	19	20	21	22
1924	23	24	25	26	27	28	29	30	31	32	33	34
1925	35	36	37	38	39	40	41	42	43	44	45	1
1926	2	3	4	5	6	7	8	9	10	11	12	13
1927	14	15	16	17	18	19	20	21	22	23	24	25
1928	26	27	28	29	30	31	32	33	34	35	36	37
1929	38	39	40	41	42	43	44	1	2	3	4	5
1930	6	7	8	9	10	11	12	13	14	15	16	17
1931	18	19	20	21	22	23	24	25	26	27	28	29
1932	30	31	32	33	34	35	36	37	38	39	40	41
1933	42	43	44	45	..	..	..	..	..	..	..	..

In Fig. 6, all the curves are reduced to a common basis of percentage of monthly averages.

TABLE 6.—SMOOTHING WEATHER-BUREAU AVERAGE CURVE.

Month No.	Average deviation, in inches.	First reduction.			Second reduction.			Equivalent percentage of mean.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	+1.48							
		+5.13	+9.53	+2.38	+4.65	+10.22	+2.56	138.8
1	+3.65							
		+3.93	+9.06	+2.27	+3.33	+7.98	+2.00	130.3
2	+0.28							
		+0.30	+4.23	+1.06	+1.34	+4.67	+1.17	117.7
3	+0.02							
		+0.84	+1.14	+0.28	+0.67	+2.01	+0.50	107.6
4	+0.82							
		+0.71	+1.55	+0.39	+0.72	+1.39	+0.35	105.3
5	−0.11							
		+0.60	+1.31	+0.33	−0.13	+0.59	+0.15	102.3
6	+0.71							
		−2.46	−1.86	−0.46	−1.98	−2.11	−0.53	92.0
7	−3.17							
		−3.60	−6.06	−1.52	−2.55	−4.53	−1.13	82.9
8	−0.43							
		−0.51	−4.11	−1.03	−1.53	−4.08	−1.02	84.5
9	−0.08							
		−1.48	−1.99	−0.50	−1.58	−3.11	−0.78	88.2
10	−1.40							
		−2.82	−4.30	−1.08	−2.47	−4.05	−1.01	84.7
11	−1.42							
		−2.75	−5.57	−1.39	−1.97	−4.44	−1.11	83.2
12	−1.33							
		+0.44	−2.31	−0.58	−0.13	−2.10	−0.52	92.1
13	+1.77							
		+1.36	+1.80	+0.45	+1.27	+1.14	+0.28	104.2
14	−0.41							
		+1.92	+3.28	+0.82	+1.38	+2.65	+0.66	110.0
15	+2.33							
		+0.32	+2.24	+0.56	−1.33	+0.05	+0.01	100.2
16	−2.01							
		−3.40	−3.08	−0.77	−1.57	−2.90	−0.72	89.1
17	−1.39							
		−3.79	−7.18	−1.80	−3.44	−5.01	−1.25	81.0
18	−2.40							
		−2.76	−6.55	−1.64	−2.84	−6.28	−1.57	76.2
19	−0.36							
		−2.04	−4.80	−1.20	−1.80	−4.64	−1.16	82.4
20	−1.68							
		−0.36	−2.40	−0.60	−0.24	−2.04	−0.51	91.2
21	+1.32							
		+1.79	+1.43	+0.36	+0.61	+0.37	+0.09	101.4
22	−0.47							
		−0.79	+1.00	+0.25	−0.28	+0.33	+0.08	101.2
23	−1.26							
		−1.33	−2.12	−0.53	−0.94	−1.22	−0.30	95.5
24	−0.07							
		−0.30	−1.63	−0.41	−0.42	−1.36	−0.34	94.8
25	−0.23							
		+0.25	−0.05	−0.01	+0.14	−0.28	−0.07	98.9
26	+0.48							
		+0.36	+0.61	+0.15	+0.07	+0.21	−0.05	99.2
27	−0.12							
		−0.67	−0.31	−0.08	−0.14	−0.07	−0.02	99.7
28	−0.55							
		+0.42	−0.25	−0.06	−0.26	−0.40	−0.10	98.5
29	+0.97							
		−1.23	−0.81	−0.20	−1.39	−1.65	−0.41	93.8
30	−2.20							
		−3.54	−4.77	−1.19	−2.62	−4.01	−1.00	84.8
31	−1.34							
		−2.17	−5.71	−1.43	−2.55	−5.17	−1.23	80.4
32	−0.83							
		−2.29	−4.46	−1.12	−2.14	−4.69	−1.17	82.3
33	−1.46							
		−1.81	−4.10	−1.02	−2.06	−4.20	−1.05	84.1
34	−0.35							
		−2.33	−4.14	−1.04	−2.03	−4.09	−1.02	84.5

TABLE 6.—(*Continued*).

Month No.	Average deviation, in inches.	First reduction.			Second reduction.			Equivalent percentage of mean.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
35	−1.98	−1.63	−3.96	−0.99	−1.59	−3.62	−0.90	86.4
36	+0.35	−0.78	−2.41	−0.60	−1.39	−2.98	−0.74	88.8
37	−1.13	−2.37	−3.15	−0.79	−1.42	−2.81	−0.70	89.4
38	−1.24	−0.14	−2.51	−0.63	+0.87	−0.55	−0.14	97.9
39	+1.10	+6.15	+6.01	+1.50	+5.20	+6.07	+1.52	123.2
40	+5.05	+8.63	+14.78	+3.70	+8.11	+13.31	+3.33	150.5
41	+3.58	+9.01	+17.64	+4.41	+8.75	+16.86	+4.22	164.0
42	+5.43	+8.35	+17.36	+4.34	+7.53	+16.28	+4.07	161.7
43	+2.92	+4.40	+12.75	+3.19	+5.57	+13.10	+3.28	151.3
40	+1.48		+ 9.53	+2.38				

Discussion of Fig. 6.—In Fig. 6 the effect of the 3.7-year cycle in all the records studied is presented graphically. All records show a marked maximum at Month 41 or 42, and the general tendency, from about Month 8 to Month 36, is to be below the mean. It may be stated that the exact length of the period or interval between rainfall peaks is more uncertain than the fact of recurrent wet periods at about 4-year intervals. This tends to spread the curves which represent a long term, and to lower their maxima. It will also be noticed that a period of about 7 months is indicated by the curves of Fig. 6.

Strength of Evidence of 3.7-Year Cycle.—The combined evidence of the curves in Fig. 6 is believed to be proof of the existence of a cyclic influence in Hawaiian rainfall, and of the possibility of its prediction.

The usual criterion for the statistical proof of such a cycle is as follows:

Let the $N = n\,p$ months of the record be arranged in tabular form as shown:

a_1	a_2	a_3	.....	a_n
b_1	b_2	b_3	.....	b_n
*	*	*		*
n_1	n_2	n_3	.....	n_n
Means of columns m_1	m_2	m_3	.....	m_n

in which a_1 to n_p are the monthly records for the n cycles of p months each.

Let σ_0 = the standard deviation for the entire record,

$$\sigma_0 = \sqrt{\frac{\Sigma\, v^2}{n}}$$

in which v is the deviation of any month from the mean of the entire record.

TABLE 7.—East Maui.—16-Year Average of Six Stations. Percentage of Long-Term Mean.

	Average percentage.	First reduction.			Second reduction.		Third reduction.	Final, reduced to 100% average.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	151		637		2 524		9 968	148.2
1	155	306	553	1 190	2 190	4 714	8 774	130.4
2	92	247	447	1 000	1 870	4 060	7 668	114.0
3	108	200	423	870	1 738	3 608	7 109	105.7
4	115	223	445	868	1 763	3 501	7 023	104.4
5	107	222	450	895	1 759	3 522	6 943	103.2
6	121	228	414	864	1 662	3 421	6 696	99.5
7	65	186	384	798	1 613	3 275	6 528	97.0
8	133	198	431	815	1 640	3 258	6 432	95.6
9	100	233	394	825	1 539	3 179	6 093	90.5
10	61	161	320	714	1 375	2 914	5 676	84.3
11	98	159	341	661	1 387	2 762	5 682	84.4
12	84	182	385	726	1 533	2 920	6 139	91.2
13	119	203	422	807	1 686	3 219	6 694	99.5
14	100	219	457	879	1 789	3 475	6 979	103.7
15	138	238	453	910	1 715	3 504	6 632	98.6
16	77	215	352	805	1 413	3 128	5 679	84.4
17	60	137	256	608	1 138	2 551	4 824	71.7
18	59	119	274	530	1 135	2 273	4 730	70.3
19	96	155	331	605	1 322	2 457	5 341	79.4
20	80	176	386	717	1 562	2 884	6 225	92.5
21	130	210	459	845	1 779	3 341	6 965	103.5
22	119	249	475	934	1 845	3 624	7 232	107.5
23	107	226	436	911	1 763	3 608	7 059	104.9
24	103	210	416	852	1 688	3 451	6 806	101.2
25	103	206	420	836	1 667	3 355	6 619	98.4
26	111	214	411	831	1 597	3 264	6 294	93.5
27	86	197	355	766	1 433	3 030	5 734	85.2
28	72	158	312	667	1 271	2 704	5 142	76.4
29	82	154	292	604	1 167	2 438	4 748	70.6
30	56	138	271	563	1 143	2 310	4 677	69.5
31	77	133	309	580	1 224	2 367	4 890	72.7
32	99	176	335	644	1 299	2 523	5 136	76.3
33	60	159	320	655	1 314	2 613	5 305	78.8
34	101	161	339	659	1 378	2 692	5 586	83.0
		178		719		2 894		

TABLE 7.—(*Continued.*)

(1)	Average percentage. (2)	First reduction (3)	(4)	(5)	Second reduction. (6)	(7)	Third reduction. (8)	Final reduced to 100% average. (9)
35	77	202	380	797	1 516	3 118	6 012	89.3
36	125	215	417	805	1 602	3 181	6 299	93.6
37	90	173	388	774	1 579	3 264	6 445	95.8
38	83	213	386	911	1 685	3 762	7 026	104.5
39	130	312	525	1 166	2 077	4 546	8 308	123.5
40	182	329	641	1 303	2 469	5 133	9 679	143.9
41	147	333	662	1 361	2 664	5 421	10 554	156.9
42	186	366	699	1 396	2 757	5 487	10 908	162.1
43	180	331	697	1 334	2 730	5 254	10 741	159.6

Let σ_n = the standard deviation for the p monthly means, m_1 to m_p,

$$\sigma_n = \sqrt{\frac{\Sigma v_m}{p}}$$

If the arrangement was entirely accidental, σ_n would be expected to equal $\frac{\sigma_0}{\sqrt{n}}$. Therefore,

$$\frac{\sigma_n}{\frac{\sigma_0}{\sqrt{n}}} = \frac{\sqrt{n}\,\sigma_n}{\sigma_0}$$

is the ratio of the standard deviation of the series of means to that expected by the law of probabilities.

This ratio itself has a probable error approximately proportional to that of σ_n, as the probable error of σ_0 is relatively small. This may be taken as:

$$e = \frac{\sigma_n \sqrt{n}}{\sigma_0 \sqrt{2p}}$$

If $\frac{\sqrt{n}\,\sigma_n}{\sigma_0} - 1$ is greater than two or three times e, the existence of a periodic law may be regarded as extremely probable. If it is less, there is a Scotch verdict of "not proven" by the magnitude of σ_n. It is evident that, no matter how valid the periodic influence, its statistical proof is impossible until N may be made sufficiently large.

Tables 9 and 10 show functions of some of the records, the smoothed curves of which are plotted in Fig. 6.

In no case is $n\,p$ great enough to give satisfactory proof that the periodic tendency is not accidental. However, the fact that increasing the value of n

TABLE 8.—Long-Term Record, by Frequency Index.

Month No.	Average frequency index.	Process of smoothing.						Smoothed record.	Reduced to percentage of mean.
0	55.7								
		113.2							
1	57.5		238.0						
		115.8		467.0					
2	58.3		229.0		968.0				
		100.9		445.7		1 881			
3	42.6		216.7		912.7		3 874	60.5	113.4
		93.4		411.0		1 769			
4	50.8		194.3		856.7		3 650	57.0	108.0
		110.6		398.3		1 666			
5	59.8		204.0		809.3		3 435	53.7	101.3
		119.6		434.2		1 642			
6	59.8		230.2		832.5		3 308	51.7	99.0
		112.9		462.7		1 729			
7	53.1		232.5		896.9		3 371	52.7	100.0
		116.7		462.1		1 822			
8	63.6		229.6		924.8		3 551	55.5	105.7
		108.1		454.4		1 841			
9	44.5		224.8		916.5		3 663	57.2	108.6
		91.3		424.2		1 795			
10	46.8		199.4		878.6		3 636	56.8	107.1
		92.2		382.9		1 686			
11	45.4		183.5		807.1		3 481	54.4	103.2
		73.6		349.3		1 539			
12	28.2		165.8		732.2		3 225	50.4	97.7
		78.6		318.0		1 399			
13	50.4		152.2		667.3		2 938	45.9	92.3
		100.2		331.0		1 316			
14	49.8		178.8		649.0		2 715	42.4	87.9
		116.1		395.1		1 375			
15	66.3		216.3		726.1		2 691	42.0	86.8
		117.0		449.4		1 571			
16	50.7		233.1		844.5		2 946	46.0	92.5
		90.5		440.6		1 734			
17	39.8		207.5		890.0		3 305	51.6	99.0
		79.0		377.0		1 708			
18	39.2		169.5		817.6		3 442	53.8	101.3
		99.7		348.2		1 543			
19	60.5		178.7		725.2		3 251	50.8	97.9
		93.7		372.1		1 445			
20	33.2		193.4		720.3		2 988	46.7	93.1
		98.9		386 0		1 478			
21	65.7		192.6		758:1		2 923	45.7	91.9
		113.0		404.5		1 549			
22	47.3		211.9		790.5		3 027	47.3	93.7
		98.0		422.9		1 618			
23	50.7		211.0		827.4		3 167	49.5	96.7
		99.9		408.9		1 659			
24	49.2		197.9		831.8		3 277	51.2	98.5
		90.7		388.5		1 629			
25	41.5		190.6		797.4		3 288	51.4	98.6
		92.4		373.7		1 560			
26	50.9		183.1		762,2		3 189	49.8	96.9
		111.2		386.7		1 523			
27	60.3		203.6		760.4		3 083	48.2	95.2
		105.8		420.6		1 568			
28	45.5		217.0		807.3		3 091	48.3	95.4
		106.1		428.9		1 657			
29	60.6		211.9		849.5		3 225	50.4	97.7
		98.7		416.7		1 695			
30	38.1		204.8		845.6		3 352	52.4	99.8
		88.4		391.9		1 654			
31	50.3		187.1		808.6		3 349	52.3	99.7
		99.4		374.9		1 575			
32	49.1		187.8		766.8		3 229	50.5	97.5
		88.9		376.1		1 518			
33	39.8		188.3		751.0		3 093	48.3	95.3
		91.0		368.2		1 495			
34	51.2		179.9		744.3		3 013	47.1	93.5
		106.8		377.7		1 490			
			197.8		745.9		2 985	46.6	92.9
				408.6		1 532			
					786.3		3 022	47.2	93.7
						1 613	3 145	49.1	95.2

TABLE 8.—(*Continued.*)

Month No.	Average frequency index.	Process of smoothing.						Smoothed record.	Reduced to percentage of mean.
35	55.6	104.0	210.8	418.3	826.9	1 666	3 279	51.2	98.6
36	48.4	103.5	207.5	420.8	839.1	1 688	3 354	52.4	99.8
37	55.1	109.8	213.3	427.9	848.7	1 714	3 402	53.2	100.7
38	54.7	104.8	214.6	437.2	865.1	1 764	3 478	54.3	102.0
39	50.1	117.8	222.6	461.6	868.8	1 843	3 607	56.4	106.7
40	67.6	121.2	239.0	482.9	941.5	1 932	3 775	59.0	111.2
41	53.6	122.7	243.9	504.8	987.7	2 016	3 948	61.7	115.5
42	69.1	138.2	260.9	523.9	1 028.7	2 054	4 070	63.6	117.6
43	69.1	124.8	263.0	501.0	1 024.9	1 993	4 047	63.2	117.0

by using larger groupings increases the value of $\frac{\sqrt{n}\,\sigma_n}{\sigma_0}$, indicates a probability that, with longer records, this proof may be obtained.

Correlation with Solar Phenomena

To determine if possible how far measurable and recognized solar changes account for the variations in Hawaiian rainfall, use was made of data on the pyrheliometric average of insolation and sun-spot numbers furnished by Professor H. H. Kimball, of the Weather Bureau. The correlations found are given in Table 13, and Fig. 4. The following facts appear to be significant:

1.—In nearly every case the greatest correlation is obtained between Hawaiian rainfall and the rate of change of the solar phenomenon, with little regard for sign, and in many cases there is no correlation between the direct value of the solar phenomenon and the rainfall.

TABLE 9.

Record.	Years.	p.	n.	σ_0	σn	$\frac{\sqrt{n}\,\sigma n}{\sigma_0}$	e.
East Maui 44 months	16	44	4.36	78.4	32.50	0.862	±0.092
" " smoothed curve	16	44	13.95	78.4	23.67	1.125	±0.120
" " grouped by 11 months	16	4	47.9	78.4	14.87	1.314	±0.464
Ten-station average 44 months	37	44	10	28·43	9.54	1.06	±0.08
" " smoothed curve	37	44	32	28.43	4.846	0.98	±0.10
Ten-station average grouped by 11 months	37	4	110	28.43	3.152	1.163	±0.41

2.—The yearly records give a much greater value of the correlation coefficient, although the probable error is also greater.

TABLE 10.—Functions of Records for Which Curves are Plotted on Fig. 6.

Month No.	Percentage of Average Rainfall.			
	Weather Bureau average.	East Maui.	36-year 10-station,	16-year leeward.
0	138.8	148.2	113.4	
1	130.3	130.4	108.0	
2	117.7	114.0	101.3	
3	107.6	105.7	99.0	94
4	105.3	104.4	100.0	
5	102.3	103.2	105.7	
6	92.0	99.5	108.6	
7	89.9	97.0	107.1	
8	84.5	95.6	103.2	84
9	88.2	90.5	97.7	
10	84.7	84.3	92.3	
11	83.2	84.4	87.9	
12	92.1	91.2	86.8	
13	104.2	99.5	92.5	79
14	110.0	103.7	99.0	
15	100.2	98.6	101.3	
16	89.1	84.4	97.9	
17	81.0	71.7	93.1	
18	76.2	70.3	91.9	76
19	82.4	79.4	93.7	
20	91.2	92.5	96.7	
21	101.4	103.5	98.5	
22	101.2	107.5	98.6	
23	95.5	104.9	96.9	87
24	94.8	101.2	95.2	
25	98.9	98.4	95.4	
26	99.2	93.5	97.7	
27	99.7	85.2	99.8	
28	98.5	76.4	99.7	96
29	93.8	70.6	97.5	
30	84.8	69.5	95.3	
31	80.4	72.7	93.5	
32	82.3	76.3	92.9	
33	84.1	78.8	93.7	102
34	84.5	83.0	95.2	
35	86.4	89.3	98.6	
36	88.8	93.6	99.8	
37	89.4	95.8	100.7	
38	97.9	104.5	102.0	136
39	123.2	123.5	106.7	
40	150.5	143.9	111.2	
41	164.0	156.9	115.5	
42	161.7	162.1	117.6	144
43	151.3	159.6	117.0	

3.—The best correlation found was between the rainfall and the change in the solar phenomena to the next year, that is, a lag of 6 months in the solar record.

4.—Both the monthly records yield average curves that are double-humped with a maximum rainfall at an intermediate rate of change in both directions and minima near zero change, and also for excessively large rates of change.

5.—In comparing Hawaiian rainfall and the rate of change of sun-spot numbers and radiation intensities, although a close correlation was found between the rainfall and each of the derivative solar functions, practically no relation existed between the two solar derivatives themselves.

Long-Term Cycles.—By removing the effect of the 3.7-year cycle from the 44-year record in a similar manner to that used with the annual cycle, Figs. 7 and 8 were obtained, indicating that the cycles of 11.1 years (corresponding to the sun-spot period) and 33 years, which have been observed by other students of precipitation, are present in Hawaiian rainfall.

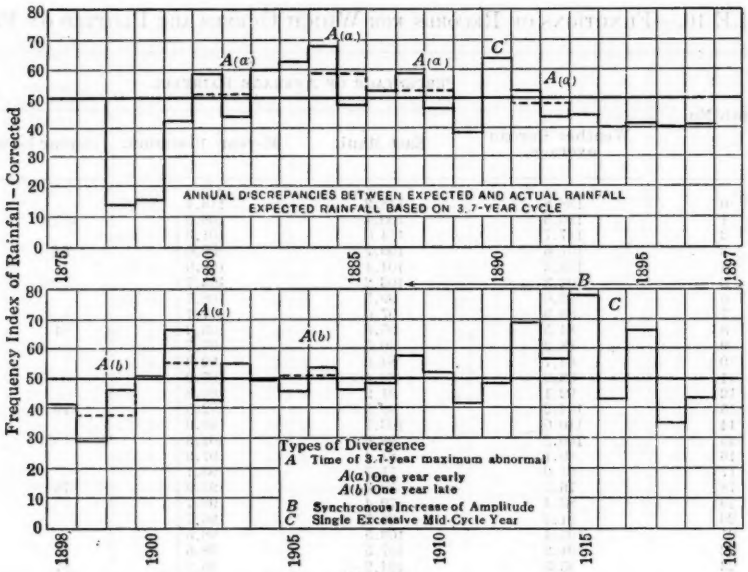


FIG. 7.

Fig. 7 is derived from the long-term record of Fig. 1 by plotting the average of the twelve monthly frequency indices corrected by the difference between 50 and the average of the twelve corresponding indices from the smoothed column of Table 7. This correction removes the effect of the 3.7-year cycle, and allows a study of residual deviations of longer term. The indications of Fig. 7 are that three types of irregular divergence exist:

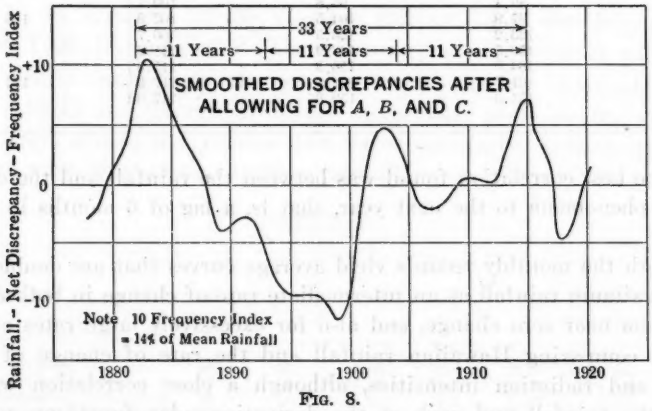


FIG. 8.

(*A*).—An abnormal shifting of the time of expected 3.7-year maxima, so that their effects occur a year too soon or a year too late.

(*B*).—At certain times notably from 1910 to 1920, the amplitude of the 3.7-year swing is considerably increased, and its period perhaps shortened.

(*C*).—In 1890 and 1916, heavy rainfall occurred at times of mid-cycle, when a low rainfall was expected. No cyclic interpretation is evident.

Fig. 8 is redrawn from Fig. 7 by eliminating the effects, A, B, and C, and enlarging the scale so as to show distinctly the 11.1-and 33-year cycles.

Forecasting.—It is evident that systematic variations are the joint effect of the 33-year, 11.1-year, 3.7-year, and annual cycles. Therefore, for a given future month, the average probability for its rainfall at a given station is $A \times B \times C$, in which A is the long-term station average for the month in question (annual cycle), B is the factor corresponding to its cycle number (3.7-year cycle, Table 5), as determined by records of similar stations (Fig. 6), and C is a factor which takes into account the position in the 11.1-year and 33-year cycles (Fig. 8). It is probable that a fourth factor should be used to allow for a systematic increase or decrease in the amplitude of the 3.7-year fluctuation.

It should be observed that this product, $A \times B \times C$, is the average probability only, and that accidental effects will cause the total probability to be as illustrated in Fig. 9. The median probability is that rainfall for which the chances are equal that it will be exceeded or that it will not be exceeded. For Hawaiian records, this is generally considerably less than the average probability.

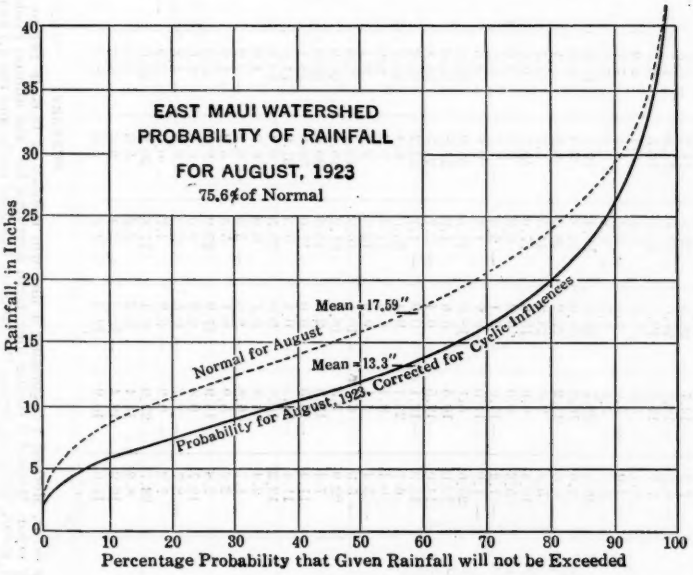


FIG. 9.

A Test of the Forecast Method.—As a test of the method outlined, Table 11 was prepared from 10 years' records for the East Maui water-shed, using the factors, A and B, but neglecting C. B was taken from the smoothed curve for the ten-station composite, and was not corrected to apply to the East Maui record, where the amplitude appears to be greater. Both these omissions are unfavorable to the suggested method.

TABLE 11.—EAST MAUI RAINFALL.—AVERAGE DEPARTURES ON THREE BASES OF COMPUTATION, 1911-1920.

		Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Long-term.	Average estimate 1	11.55	17.59	18.41	20.25	15.65	12.84	15.52	17.78	13.30	14.36	18.66	18.82	194.73
1920.....	Rain	4.3	3.8	45.00	7.2	1.4	7.5	11.0	14.2	12.1	14.0	18.2	14.8	153.5
	Dep. 1	11.3	—11.8	29.4	— 8.4	—14.2	— 8.1	— 4.6	— 1.4	— 3.5	— 1.6	+ 2.6	— 0.8	97.7
	" 2	— 7.2	—13.7	26.5	—13.0	—14.2	— 5.3	— 4.5	— 3.6	— 1.2	— 0.4	— 0.5	— 4.0	94.1
	" 3	— 6.5	—13.3	27.0	—12.8	—13.8	— 4.7	— 3.7	— 3.2	— 1.2	— 0.4	— 0.3	— 3.3	90.2
1919.....	Rain	7.7	12.6	19.5	11.8	10.5	5.7	15.1	16.5	11.2	13.2	4.9	11.0	139.6
	Dep. 1	8.1	3.0	3.8	3.8	5.1	9.9	0.5	0.9	4.4	2.4	10.8	4.6	57.3
	" 2	— 4.0	— 5.0	1.0	— 8.4	— 5.1	— 7.1	— 0.4	— 1.3	— 2.1	— 1.2	— 13.8	— 7.8	57.2
	" 3	— 5.1	— 5.5	1.5	— 6.8	— 3.3	— 5.5	0.8	— 1.1	— 2.2	— 0.8	—12.5	— 6.3	51.4
1918.....	Rain	18.1	28.7	45.6	68.4	19.5	17.9	29.8	23.5	5.9	7.7	14.3	25.4	304.8
	Dep. 1	2.6	13.2	30.0	52.8	3.9	2.3	14.2	7.9	9.7	7.9	1.4	9.8	155.7
	" 2	6.6	11.2	27.2	48.2	3.9	5.1	14.3	5.7	— 7.4	— 6.7	— 4.4	6.6	147.3
	" 3	5.8	9.0	24.2	44.5	1.2	3.7	13.0	5.4	— 7.3	— 6.7	— 5.5	4.9	131.2
1917.....	Rain	10.4	6.6	15.3	17.9	13.9	9.3	7.1	8.6	4.9	4.0	13.4	8.1	119.5
	Dep. 1	5.2	9.0	0.3	2.3	1.7	6.3	8.5	7.0	10.7	11.6	2.3	7.5	72.4
	" 2	— 1.2	—11.0	— 3.1	— 2.3	— 7.7	— 3.5	— 8.4	— 9.2	— 8.4	—10.4	— 5.3	—10.7	81.2
	" 3	— 1.2	—11.0	— 2.9	— 1.5	— 0.8	— 2.6	— 7.5	— 8.3	— 8.3	—10.4	— 5.5	—11.1	71.1
1916.....	Rain	25.6	5.1	18.0	17.5	36.9	25.6	17.1	22.8	16.8	16.1	21.0	34.0	256.5
	Dep. 1	10.0	10.5	3.6	1.9	21.8	10.0	1.5	2.0	2.7	2.0	5.3	18.4	89.7
	" 2	14.0	—12.5	— 0.4	— 2.7	21.3	12.8	1.6	5.0	3.5	1.7	2.3	15.2	93.0
	" 3	13.9	—12.1	0.9	— 1.1	22.2	13.2	1.9	5.2	3.9	2.5	3.3	15.5	95.7
1915.....	Rain	6.1	23.7	9.4	33.0	6.4	16.2	15.8	8.7	20.8	28.2	36.1	18.8	223.2
	Dep. 1	9.4	8.1	6.2	17.4	9.2	0.6	0.2	6.9	5.2	12.6	20.4	3.2	99.4
	" 2	— 5.4	6.1	— 9.0	12.8	— 9.2	3.4	0.3	— 9.1	7.5	13.8	17.4	0.0	94.0
	" 3	— 5.3	6.1	—10.1	10.9	—10.7	3.0	0.6	— 7.6	9.1	15.7	18.9	— 0.2	98.2
1914.....	Rain	22.9	2.3	18.4	31.2	49.4	28.9	33.9	42.8	39.7	19.0	27.2	17.9	333.6
	Dep. 1	7.4	13.3	2.8	15.6	33.8	13.3	18.3	27.2	14.1	3.4	11.5	2.3	163.0
	" 2	11.4	—15.3	0.0	11.0	33.8	16.1	18.4	25.0	26.4	4.6	8.5	0.9	171.4
	" 3	11.4	—15.5	— 0.4	9.5	31.9	14.0	15.6	21.8	24.9	3.0	7.0	— 1.1	156.1
1913.....	Rain	13.2	10.8	9.9	25.5	9.4	10.8	8.1	12.9	6.1	9.9	34.7	20.3	171.6
	Dep. 1	2.4	4.8	5.7	9.9	6.2	4.8	7.5	2.7	9.5	5.7	19.0	4.7	82.9
	" 2	1.7	— 6.8	— 8.6	— 5.3	— 6.2	— 2.0	— 7.4	— 4.9	— 7.2	— 4.5	16.0	1.5	72.1
	" 3	2.2	— 5.9	— 8.1	5.3	— 6.2	— 1.9	— 6.8	— 3.8	— 6.3	— 3.6	17.0	1.6	68.7
1912.....	Rain	2.8	15.9	17.9	25.2	9.3	8.0	8.3	8.3	6.6	16.9	16.7	32.6	168.5
	Dep. 1	12.7	0.3	2.3	9.6	6.3	7.6	7.3	7.3	9.0	1.3	1.1	17.0	81.8
	" 2	— 8.7	— 1.7	— 0.5	5.0	— 6.3	— 4.8	— 7.2	— 9.5	— 6.7	2.5	— 2.0	13.8	68.7
	" 3	— 7.2	— 0.3	— 0.3	4.7	— 6.0	— 3.9	— 6.0	— 8.4	— 6.3	2.9	— 1.8	14.3	62.1
1911.....	Rain	23.7	29.9	6.4	17.8	20.9	22.0	12.2	21.2	36.8	9.5	18.2	14.2	232.8
	Dep. 1	8.2	14.4	9.2	2.2	5.3	5.4	3.4	5.6	21.2	6.1	2.6	1.4	85.0
	" 2	12.2	12.4	—12.0	— 2.4	5.3	9.2	— 3.3	3.4	23.5	— 4.9	— 0.5	— 4.6	93.7
	" 3	11.0	10.9	—12.2	— 2.2	5.3	8.4	— 4.7	1.7	23.1	— 4.4	1.0	— 2.4	87.3

SUMMARY:	Inches.	Percentage.
Average monthly error, Method 1 (assuming a uniform rainfall of 15.65 in per month)	8.207	52.5
Average monthly error, Method 2 (using long term averages for the annual cycle)	8.106	51.7
Average monthly error, Method 3 (using the long-term annual cycle and the 3.7 year cycle for the 36-year record)	7.600	48.5
Improvement of Method 2 over Method 1	0.101	0.8
" " " 3 " " 2	0.506	3.2
Number of times 3.7-year cycle improves	76	....
" " " 3.7 " " is on wrong side	33	....
" " " 3.7 " " causes no change	11	....
Proportion of improvement		67.9

TABLE 12.—FUTURE PROBABILITY OF RAINFALL, EAST MAUI WATERSHED.

Month.	Cycle no	Average for month, in inches. *A*	*B*	*C*	Average expectation ($A \times B \times C$), in inches.	Median expectation 0.88 ($A \times B \times C$), in inches.
1922.						
Jan.	43	11.55	1.60	1.04	19.2	16.9
Feb.	44	17.59	1.48	1.04	27.1	23.8
Mar.	1	18.41	1.30	1.04	24.9	21.9
Apr.	2	20.35	1.14	1.04	24.0	21.1
May.	3	15.65	1.06	1.05	17.4	15.3
June.	4	12.84	1.04	1.05	14.0	12.3
July.	5	15.52	1.03	1.05	16.8	14.8
Aug.	6	17.78	1.00	1.05	18.7	16.4
Sept.	7	13.30	0.97	1.05	13.5	11.9
Oct.	8	14.36	0.96	1.05	14 5	12.8
Nov.	9	18.66	0.90	1.06	17.8	15.6
Dec.	10	18.82	0.84	1.06	16.7	14.7
1923.						
Jan.	11	11.55	0.84	1 06	10.3	9.0
Feb.	12	17.59	0.91	1.06	17.0	14.9
Mar.	13	18.41	1.00	1.06	19.5	17.1
Apr.	14	20.25	1.04	1.06	22.3	19.6
May.	15	15.65	0.99	1.06	16.4	14.4
June.	16	12.84	0.84	1.06	11.4	10.1
July.	17	15.52	0.72	1.06	11.8	10.4
Aug.	18	17.78	0.70	1.07	13.3	11.7
Sept.	19	13.30	0.79	1.07	11.2	9.9
Oct.	20	14.36	0.92	1.07	14.1	12.4
Nov.	21	18.66	1.04	1.07	20.8	18.3
Dec.	22	18.82	1.08	1.07	21.7	19.1

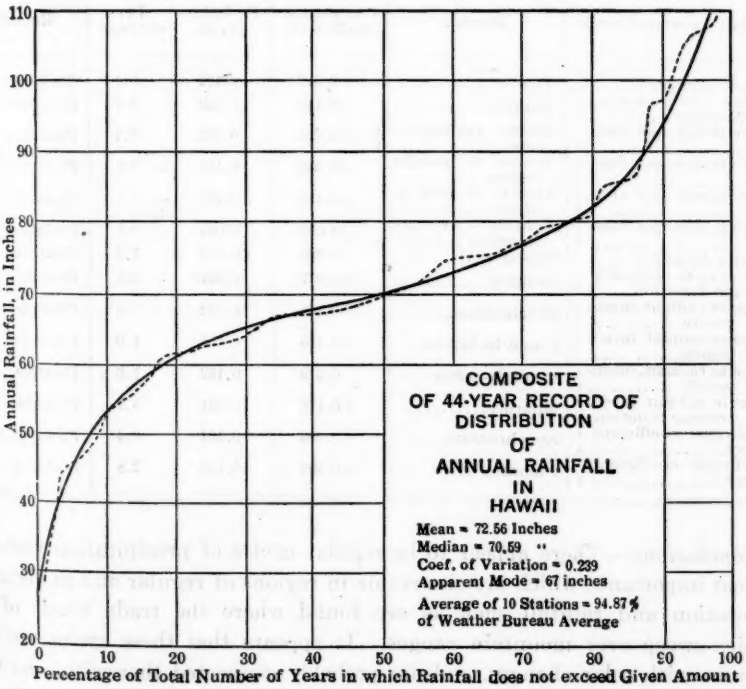


FIG. 10.

In Table 11 a comparison is made between the observed rainfall and three *a priori* estimates. Estimate 1 assumes a uniform monthly rainfall of 15.65 in. Estimate 2 assumes that the rainfall is affected by the annual cycle (Fig. 3), and corresponds to the long-term monthly averages. These are given in the first line of figures in Table 11. Estimate 3 assumes that the rainfall is affected by the annual cycle and by the 3.7-year cycle, as determined from the ten-station record. The body of Table 11 gives the observed rainfall and its departure from each of the three estimates. The average departure for each method of estimating was then computed, and is tabulated in the second part of Table 11. The proposed method of forecasting probable rainfall (Estimate 3) is an improvement over the use of monthly averages in two-thirds of the number of months, and reduces the average error by 6½% of its original value. It is significant that the improvement by the use of the 3.7-year cycle is greater than that of using the well-recognized annual cycle, or, in other words, that it is not as wise to expect a wet month in Position 18 of the 3.7-year cycle as to expect a wet October. The importance of such an improvement in the knowledge of rainfall probabilities cannot be over-estimated, and its practical applications are obvious.

TABLE 13.—Summary of Correlations Between Hawaiian Rainfall and Solar Phenomena.

Rainfall correlated with:	Period.	Correlation coefficient.	Probable error.	Per-centage.	Remarks.
Sun-spot numbers	Yearly	−0.012	0.164	0.1	Probably none
Sun-spot numbers, average function	Monthly	+0.159	0.043	3.7	Doubtful
Change in sun-spot numbers	Yearly (synchronous)	+0.070	0.163	0.4	Probably none
Change in sun-spot numbers	Yearly, 6 months before	−0.056	0.164	0.3	Probably none
Change in sun-spot numbers	Yearly, 6 months after	+0.459	0.130	3.5	Probable
Change in sun-spot numbers	Monthly — Average function	+0.092	0.044	2.1	Probably small
Radiation intensity	Yearly	−0.200	0.153	1.3	Doubtful
Radiation intensity, average function	Monthly	+0.077	0.0313	2.5	Probably small
Change in radiant intensity, yearly	Synchronous	+0.245	0.152	1.6	Possible
Change in radiant, intensity, yearly	6 months before	+0.165	0.158	1.0	Doubtful
Change in radiant, intensity, yearly	6 months after	0.249	0.152	1.6	Possible
Change in radiant, intensity, average function	Monthly	+0.176	0.031	5.7	Probable
Humphreys' coefficient, yearly	Synchronous	−0.063	0.164	0.4	Probably none
Humphreys' coefficient, yearly	6 months after	+0.399	0.140	2.8	Probable

Conclusions.—There appear to be regular cycles of precipitation variation of great importance which are observable in regions of regular and undisturbed evaporation and rainfall such as are found where the trade winds of the Tropics sweep over mountain ranges. It appears that these cycles are synchronous with solar changes, and are probably connected therewith, and that, under similar conditions, estimates of future rainfall and the extension of

existing records can be greatly improved by such a statistical study as has been made for the Hawaiian Islands.

Acknowledgments.—Especial acknowledgment must be made to the contribution of Bigelow's "Atmospheric Circulation and Radiation" and Humphreys' "Physics of the Air" to the studies presented in this paper, and to the assistance of L. H. Daingerfield, Meteorologist in charge of the Honolulu Weather Bureau Office, and Professor Kimball of the U. S. Weather Bureau in Washington, as well as to the advice and encouragement of J. H. Foss, Assoc. M. Am. Soc. C. E., Chief Engineer of the Hawaiian Commercial and Sugar Company and Maui Agricultural Company, Paia, Maui.

THE RIVER AND HARBOR PROBLEMS OF THE LOWER MISSISSIPPI*

A SYMPOSIUM

THE MOUTHS OF THE MISSISSIPPI RIVER

By Elliott J. Dent,† M. Am. Soc. C. E.

The total expenditures for the improvement of the navigable channels at the mouths of the Mississippi River, plus funds now on hand, amount to approximately $30 000 000. Only a few features of this work, that are believed to be most important and most interesting, will be discussed.

Fig. 37 is a map showing contours in this portion of the Gulf of Mexico, the depths being in fathoms. An examination of the chart shows how the 10-fathom contour has been forced out for many miles and how the outer slope of the delta in the vicinity of the present mouth of the river has already reached the 50-fathom contour. There is a sudden change in bottom slope in the vicinity of the 50-fathom depth. In the past, the delta has been advancing in relatively shallow water of slowly increasing depth. A future advance of 10 miles would put the end of South Pass at a point where the water is now 100 fathoms deep, and a 20-mile advance would reach the 400-fathom contour.

The increased depth and increased bottom slope are of the utmost importance in determining the rate of advance of the delta. In deep water it takes more material to build new land than in shallow water. A more important fact is that most of the sediment discharged by the river settles slowly through the water, and will travel much farther if the water is deep than if it is shallow. For example, a particle settling through still water at a rate of 3 in. per min. would reach the bottom in 48 min., if the water is 12 ft. deep, and would remain in suspension 16 hours if carried as far as the 40-fathom contour in that time. In the deep water off the mouth of the river, the Gulf currents are stronger than in the bays and sounds on either side of the axis of the delta, and these currents are also important agents in scattering the sedimentary load.

Fig. 38 shows the changes in land areas that have taken place since the first detailed survey, in 1838. Near the mouth of the Mississippi River, these changes are taking place at a rapid rate. The areas hatched and dotted have existed since 1838; the areas hatched, but not dotted, have settled and disappeared since 1838; and the areas dotted only have been formed since 1838.

A new railroad fill settles for a number of years, and the fresh delta deposits of the Mississippi River similarly settle and consolidate for a few centuries. At the Head of Passes, the rate of settlement is about 0.2 in. per year; at the mouth of South Pass the rate is about 0.3 in. per year and at

* Presented at the Spring Meeting of the Society at New Orleans, La., on April 18 and 19, 1923, and continued from August, 1923, *Proceedings*.

† Lt.-Col., Corps of Engineers, U. S. Army, New Orleans, La.

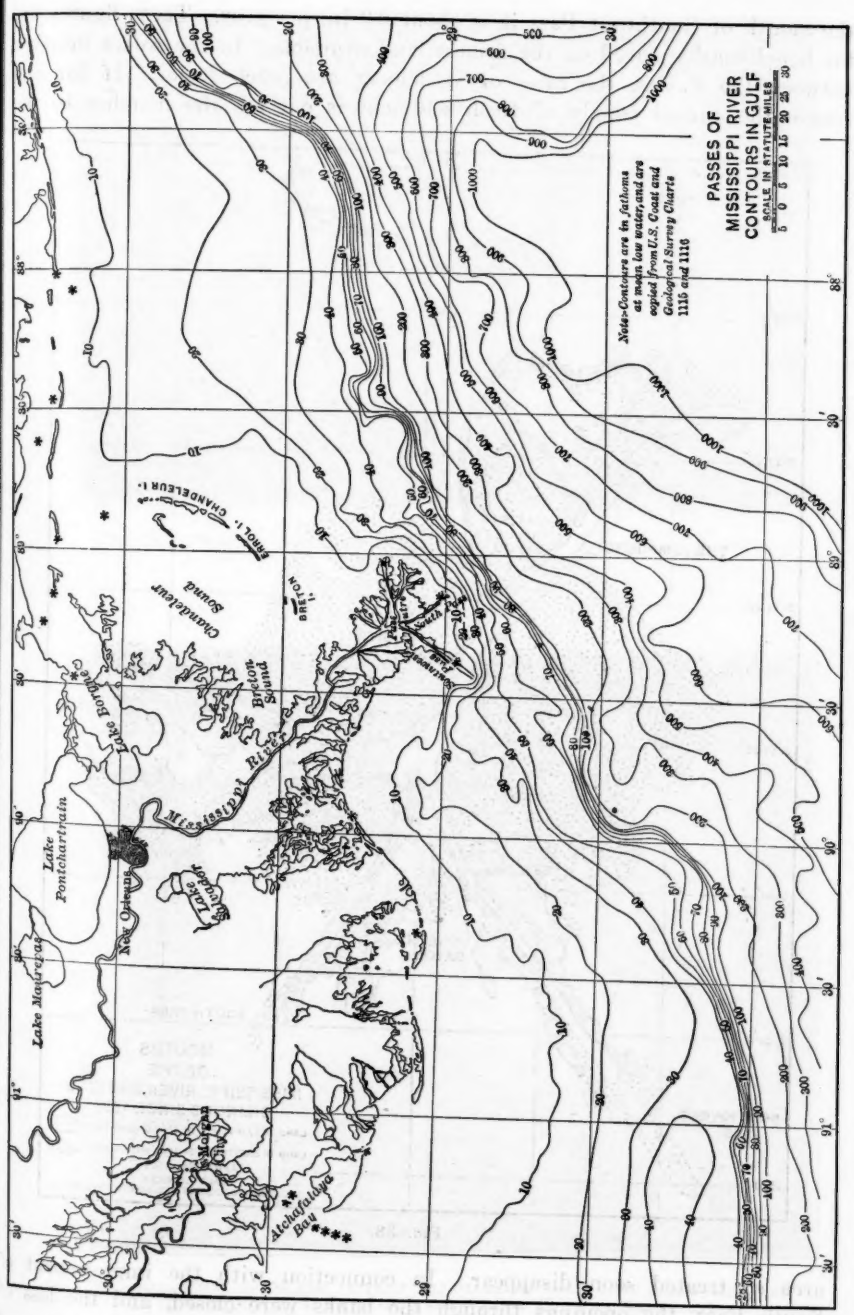

PASSES OF
MISSISSIPPI RIVER
CONTOURS IN GULF
SCALE IN STATUTE MILES
5 0 5 10 15 20 25 30
Note-Contours are in fathoms
at mean low water, and are
copied from U.S. Coast and
Geological Survey Charts
1115 and 1116
Chandeleur Sound
Breton Sound
Lake Borgne
Lake Pontchartrain
Lake Maurepas
New Orleans
Mississippi River
Atchafalaya Bay

FIG. 37.

the mouth of Southwest Pass it is about 0.9 in. per year. These figures are for bench-marks placed on the firmest land available. In the softer marshes between the Passes, the rates of settlement are much faster. If for any reason the annual supply of fresh sediment is cut off, the marshes in the

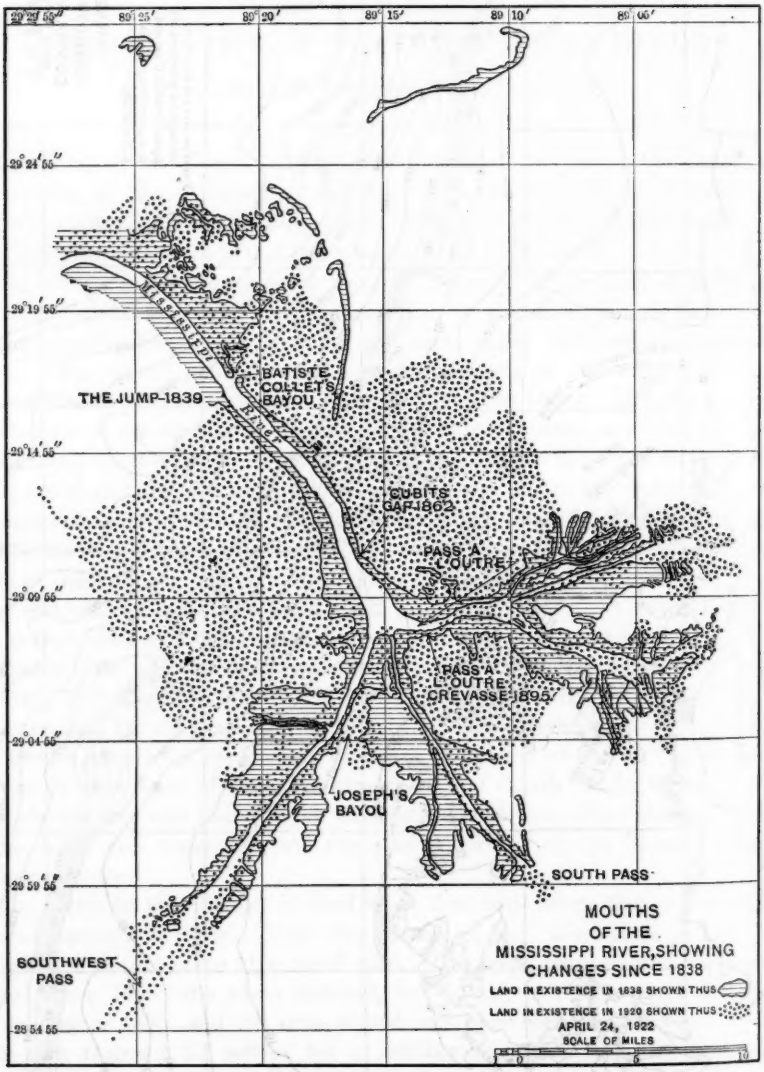


FIG. 38.

area so treated soon disappear. In connection with the improvement of South Pass, the openings through the banks were closed, and the loss of marsh area along the west bank is one of the big changes that have occurred since 1838.

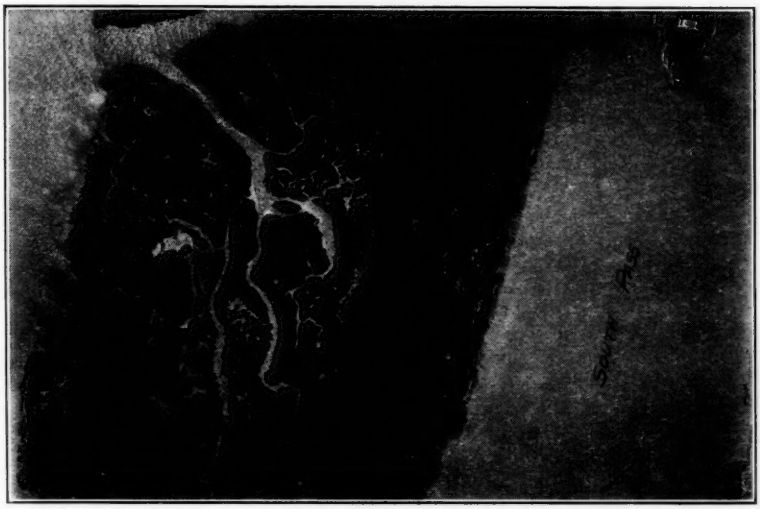


FIG. 39.—VIEW OF SUBSIDING MARSH, WITH SYSTEM OF TIDAL PONDS AND CONNECTING CHANNELS.

FIG. 40.—VIEW OF MINOR OUTLET DISCHARGING INTO A SHALLOW BAY AND ADVANCING AT A RATE OF ABOUT 1 000 FT. PER YEAR, AND ROUND ISLANDS OF MARSH CANE.

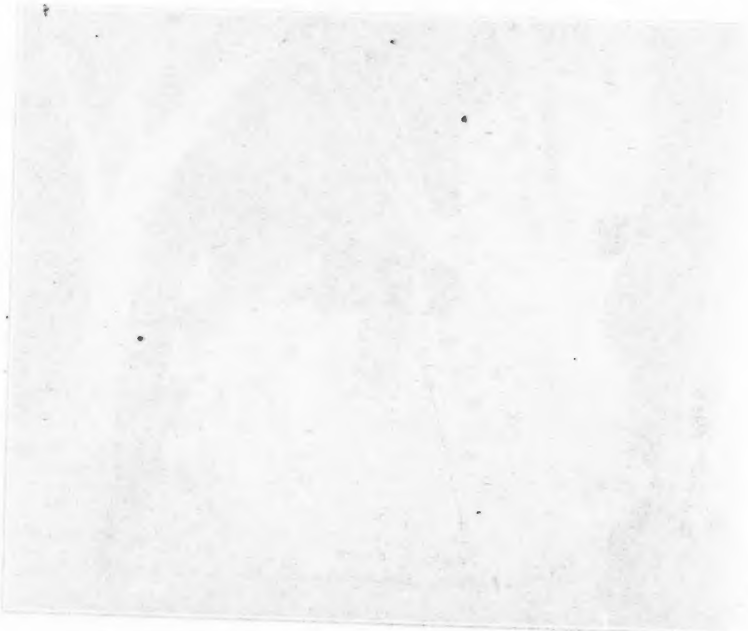


FIG. 41.—VIEW OF PART OF BANK OF SOUTH PASS, MADE THIN BY SUBSIDENCE AND WAVE AND CURRENT EROSION, DETACHED ISLANDS REPRESENT BANKS OF AN OLD BAYOU.

Fig. 38 also shows the vast areas of new land that have been formed since 1838. The sedimentary load of the river during high water may be assumed as 1 000 parts per million, or 0.1 of 1 per cent. If conditions favor sedimentation, fresh deposits of considerable depth will form in a single season. Dikes at the Head of Passes designed to reduce the velocity of flow to about one-half, have produced deposits as much as 10 ft. in depth in a single year. In the bays bordering the delta, fresh deposits more than 1 ft. in thickness are of frequent occurrence. In addition to favorable sedimentation conditions, if the depths are shallow, new marsh areas are created in a surprisingly short time.

Batiste Colet's Bayou, an outlet so small that no accurate record of its behavior has been kept, has built up its sub-delta since a survey made in 1868. The Jump broke in 1838, and has extended its delta 12 miles since that time. Cubits Gap broke in 1862, and its largest branch is now more than 8 miles in length. Pass a l'Outre Crevasse broke in 1890, and has advanced its delta at a rate of approximately ¼ mile per year. During the period since 1838, Southwest Pass has discharged more water than all the previously mentioned outlets combined, but the amount of new land that it has created is negligible.

The fresh delta deposits settle, and, as each marsh area subsides, an intricate system of tidal sloughs and ponds is created. Fig. 39 shows a subsiding marsh with such a system of ponds and connecting channels.

Fig. 40 shows a minor outlet discharging into shallow water and advancing at a rate of about 1 000 ft. per year. Note the round islands of marsh cane well in advance of the remainder of the shore line.

Fig. 41 shows a portion of the bank of South Pass that has become dangerously thin, due to subsidence and wave erosion on the bay side and current erosion on the channel side. This photograph also shows two little outlets recently cut to allow the sediment-laden water to fill in the bay and strengthen the bank. In 1922, the depth of deposit resulting from these outlets averaged about 0.5 ft. for a distance of about 2 000 ft. from the bank. The detached islands shown on Fig. 41 represent the banks of an old bayou. In 1838, a continuous marsh area covered the space between the old bayou and the bank of South Pass. Two openings, discharging about 400 sec.-ft., each, have been cut with a view to filling this portion of the bay with sediment. Fig. 40 shows the east bank of South Pass at this point. Along the west bank, wave action and settlement are controlling factors. Along the east bank, new marsh areas are forming.

A description has been given of the extent of new areas built up by the sedimentary load of the river. A brief description of the sediment itself and the manner in which it is transported may be desirable. The sediment ranges from fine sand to clay. A composite sample, made up of 235 samples taken at various depths and various localities all over the lower delta, showed that about 1% would be retained on a 50-mesh sieve and that about 75% would pass a 200-mesh sieve. The largest particles of sand passing the 200-mesh sieve will settle through still water at a rate of about 12 in. per min. It is easy to obtain samples of sand near the mouth of the river, the individual particles of which

settle as slowly as 2½ in. per min. Practically all sediment discharged by the river is so fine that it can readily be thrown into a condition of complete suspension, and probably more than 98% of the whole sedimentary load is discharged in a suspended state. The amount of sand rolled along the bottom is not sufficient to exercise a visible influence upon the form of the bars or the growth of new deposits.

Fig. 42 shows depths on the South Pass bar. When the jetties were first built—in 1878—they reached to about the 30-ft. depth. Since that time, the bar has advanced as shown until the 30-ft. depth along the axis of the jettied channel is now approximately 3 000 ft. farther seaward. It will be seen that the channel through this newly formed bar inclines sharply to the left. The sediment-laden water goes straight ahead, or rather spreads out, fan-shaped, and the channel through the bar is located where the fresh water and the sediment do not go. In this channel, the average depth of dredging is about 1 ft. per year, and the greatest shoaling is along the edge nearest the axis of the pass.

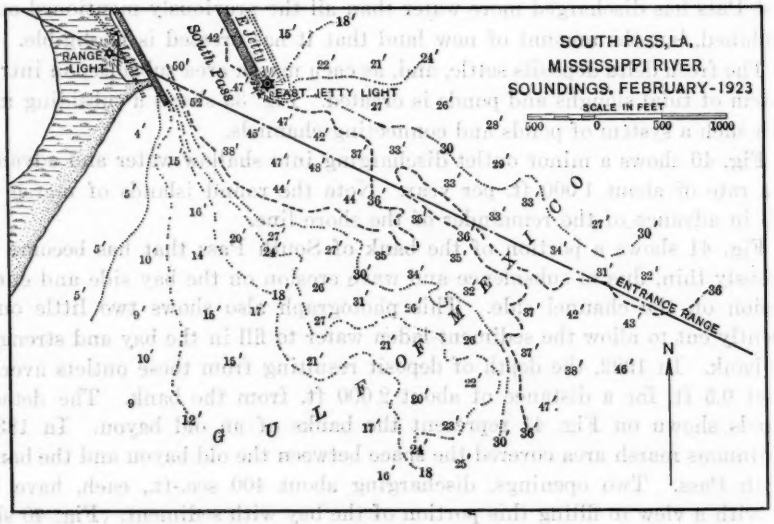


FIG. 42.

Fig. 43 illustrates the static forces set up when a stream of fresh water is discharged into a salt-water sea. The difference in specific gravity causes the fresh water to spread out into a lens-like section, while the salt water forces its way underneath. If the depth is different on the two sides, the salt water may flow in from the deep side only. If the salt water is 30 ft. deep, the fresh-water column must stand about 8 in. higher to produce equilibrium at the bottom. At South Pass, the difference in elevation of the water surfaces on the inside and outside of the east jetty is less than that required for equilibrium at a depth of 30 ft. As a result of these conditions, one should expect the fresh water to spread out fan-shaped on the surface and the salt water to flow from the east side along the bottom toward the axis of the fresh-water stream. On

the west side, where the water is less deep, the super-elevation of the fresh water may be sufficient to prevent any inflow of salt water.

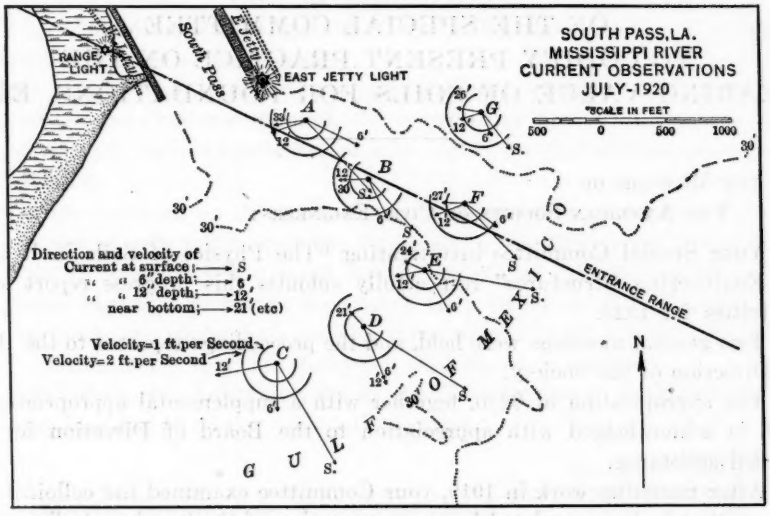


FIG. 43.

Fig. 44 shows a set of current observations on the South Pass bar. On the surface, the fresh water spreads out fan-like, as shown by the arrows representing the currents at the surface and at the 6-ft. depth, but on the bottom, in depths of 12 ft. and more, there is an inflow of salt water across the lateral channel toward the axis of the pass. These salt-water bottom currents have velocities of 1 ft. to 2 ft. per sec., and are ideally ordered to prevent the deposit in the bar channel of any sediment discharged by the pass. Any particle settling deep enough to fall under the influence of the salt-water current is promptly turned back toward the shoal water to the west of the navigable channel.

This subject is so large that only a few points can be mentioned. The engineering problem as a whole involves the maintenance of a proper distribution of flow between the three main Passes, the maintenance of such width as far as the ends of the jetties that the currents will themselves maintain the channel, and the maintenance of a channel from the ends of the jetties through the bar to deep water. A channel has been maintained through South Pass for more than 40 years, and the channel through Southwest Pass should soon be as good as or better than that through South Pass. The result of years of experience and the knowledge that has been acquired during that time have upset many old theories and have established many new principles; it is hoped that those described herein will prove of interest to the Profession.

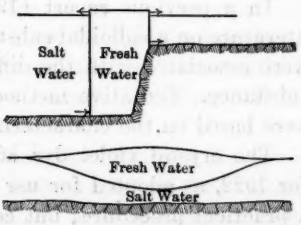


FIG. 44.

PROGRESS REPORT OF THE SPECIAL COMMITTEE TO CODIFY PRESENT PRACTICE ON THE BEARING VALUE OF SOILS FOR FOUNDATIONS, ETC.*

To the Members of
The American Society of Civil Engineers:

Your Special Committee investigating "The Physics of Soils in Relation to Engineering Structures" respectfully submits this progress report of its activities for 1922.

Two general meetings were held, and the proceedings reported to the Board of Direction of the Society.

The appropriation of $250, together with a supplemental appropriation of $40, is acknowledged with appreciation to the Board of Direction for this needed assistance.

After resuming work in 1919, your Committee examined the colloidal substance that had accumulated by the evaporation of the "wash water" from its analyses of clay. Test briquettes were made of this material, neat and mixed with Ottawa sand. The briquettes developed unusual strength, and the results were reported in the progress report for 1920. The investigation has since expanded, and is still in progress.

Recently, Dr. Milton Whitney found that the turbid flood waters of the Potomac and other rivers carry a colloidal substance in suspension. This substance possesses characteristics similar to the substance taken from the wash water of the clay analyses previously reported by your Committee. It may also be noted that the film deposited on the surface of slow sand filtration beds by raw water substantiates the thought that the origin of this substance is in the natural soil.

In a previous report (1921), your Committee submitted a review of the literature on a colloidal substance and a colloidal property. These two concepts were associated with the difficulty of separating and identifying the colloidal substance. Tentative methods for estimating the amount present in the soil were based on the characteristic property of colloidal matter—absorption.

The crystal violet dye adsorption method reported in the progress report for 1922, as adopted for use by the United States Bureau of Public Roads, is a practical procedure; but certain mineral constituents of soils may affect the results, so that further experimentation is desirable. For this purpose, the United States Bureau of Soils is investigating the ammonia-gas and the water-vapor methods. The results, when complete, will be presented in a future report.

* Presented to the Annual Meeting, January 17, 1923. Previous Progress Reports of this Special Committee have been published in *Proceedings*, Am. Soc. C. E. (Papers and Discussions), February, 1915, p. 491; March, 1916, p. 343; August, 1917, p. 1 171; August, 1920, p. 905; February, 1921, p. 9; and March, 1922, p. 523.

In the meantime, the successful separation of the colloidal substance by super-centrifuging and filtration methods has been demonstrated by the soil physicists of the U. S. Bureau of Soils.*

Your Committee is thus enabled to report that the existence of a separate colloidal substance is established.

The samples of soils and the separated colloidal substance which accompany this report are exhibited through the courtesy of Dr. Whitney, who is co-operating with your Committee, and to whom its appreciation and thanks are extended.

Further emphasis on the practical significance of this substance is indicated by samples of soils taken from the levees along the Mississippi River. Those samples which contained a large percentage of the colloidal substance were afterward identified as having been taken from levees which had proved stable during the floods, while those samples containing a small percentage had been secured from levees that had developed weaknesses or crevasses.

Reference to the following tests by the U. S. Bureau of Soils indicates the physical importance of the colloidal substance:

	Percentage of colloids.	Crushing strength, in pounds.
Sharkey clay soil from Valley Park, Miss.....	48	4 700
Fine sandy loam subsoil from Orangeburg, S. C.	21	800
Fine sandy loam subsoil from Goldsboro, N. C..	11	350

The soil specimens were 25 mm. high and 25 mm. in diameter, made up in moist condition under a pressure of 1 800 lb. per sq. in. and dried at 100° cent. The specimens were then crushed, with the comparative results given above.

In Appendix I will be found further confirmatory data relating to the physical and colloidal properties of soils. This contribution to the investigation of clays is a continuation of the work reported in 1922 by J. H. Griffith, M. Am. Soc. C. E., and was carried out by the Engineering Experiment Station of Iowa State College, Dean Anson Marston, M. Am. Soc. C. E., Director, in co-operation with the U. S. Bureau of Public Roads, Thomas H. MacDonald, Chief of the Bureau. It is due to the courtesy of these officials that this advance abstract is available. Your Committee justifies the delay in printing its report pending the receipt of this valuable, although incomplete, contribution. Your Committee here records its appreciation of this co-operation and the importance of the results.

Encouraging information has reached your Committee that similar investigations into the physics of soils are about to be undertaken by other technical institutions, which will be reported when available. It is the expectation of your Committee that such experimentation will justify the hope that results will be forthcoming at an early date for the practicing engineer.

Appendix II presents a review and conclusions by the Sub-Committee on "Codification of Information on Bearing Capacity of Soils." Observations resulting from discussion will have their influence in the final report.

* A description of the methods may be found in *Bulletin No. 1122*, of the U. S. Bureau of Soils.

Your Committee acknowledges with much appreciation the receipt of a copy of the report on Landslides from the Geo-technical Commission of the Royal State Railways of Sweden. This subject will continue to have the attention of your Committee.

Acknowledgment is also due to Mr. Jay Downer, Chief Engineer, Bronx Parkway Commission, who contributed a copy of an interesting report on earth-pressure tests recently conducted by Mr. Dudley T. Babcock at Palmer Avenue Bridge, New York, N. Y. Also to Jacob Feld, Jun. Am. Soc. C. E., for the receipt of a valuable historical review of earth pressure theories, presenting a most interesting record of the repetitive efforts of succeeding generations in covering the same subject.

If the continuation of the Committee work is approved, it is expected to continue the study already given to colloidal substance and other phases of the problem that have occupied its attention.

Your Committee can fix no date for the completion of its work, but can give assurance that when practical results are available, the facts will be presented without delay.

Respectfully submitted,

ROBERT A. CUMMINGS,

Chairman, Soils Committee,

Am. Soc. C. E.

January 17, 1923.

THE COMMITTEE:

ROBERT A. CUMMINGS, *Chairman,*
WALTER J. DOUGLAS, *Secretary,*
E. G. HAINES,
ALLEN HAZEN,
J. C. MEEM.

APPENDIX I

PROGRESS REPORT ON 1922 SOILS INVESTIGATION AT IOWA STATE COLLEGE

I.—ANNOUNCEMENT

Introduction.—The following is a synopsis of an investigation on the physics of soils, which has been conducted at Iowa State College during 1922. The work is largely a continuation of that started in 1921, in co-operation with the Soils Committee of the American Society of Civil Engineers.

Outline of Program of Research.—In the main, the research covered the following lines of special investigation: (1) The determination of the compressive strengths of soils; (2) the investigation of tensile strengths; (3) the behavior of soils under time loadings; (4) investigation of shearing strengths; and (5) special studies of deportments of colloids, in addition to the parallel correlation tests already mentioned.

II.—Compression Tests

Apparatus.—Apparatus of various types has been designed for making compression tests. One of these designed for field tests is shown in Fig. 1. The soil is tested in its natural bed, as shown in the diagram. The loaded platform is lowered on the specimen as indicated and the "back pull" measured by the dynamometer. Moisture, colloidal contents, and unit weights were obtained.

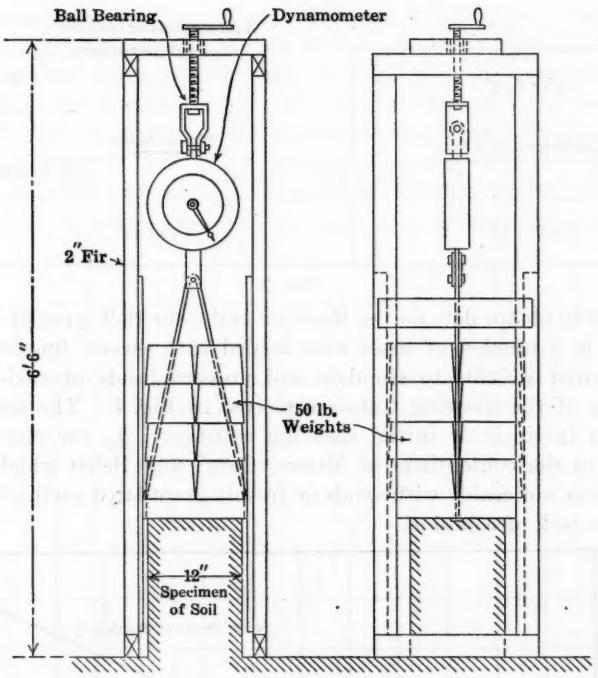


Fig. 1.

Results of Tests.—The average strength of the loams was 0.27 ton per sq. ft., the maximum value observed being 0.48 and the minimum 0.14. The clays supported an average of 0.33 ton per sq. ft., the observed maximum being 0.41 and the minimum 0.23.

There was a general tendency for the compression strengths to increase as a linear function of the colloidal contents, with, however, some scattering of observations. It seems expedient to withhold results of this part of the Committee's work until further data have been secured and a decisive law is obtained.

III.—Shear Tests

The shearing tests were made with the apparatus, shown in Fig. 2. The specimens were cut out of the solid, the "flasks" inserted in position, and the spaces between the specimens tamped with loam. Power was applied by the crank and screw and measured by readings taken on the dynamometer.

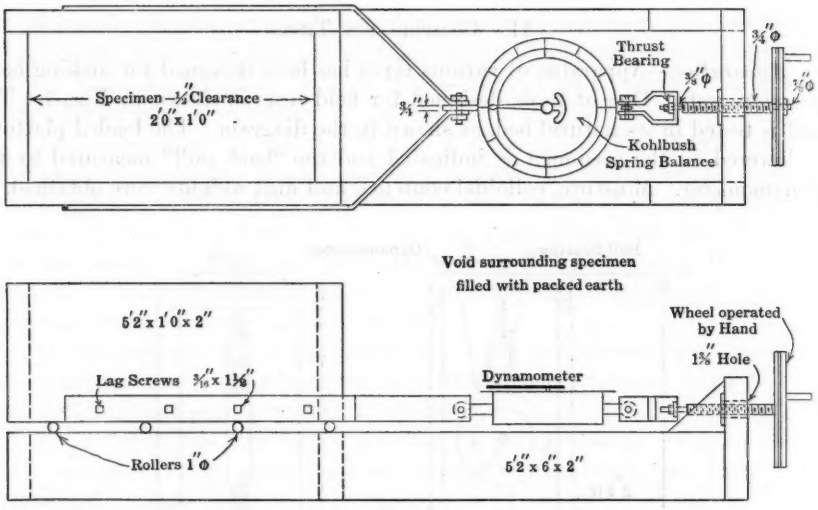


FIG. 2.

In order to obtain data on the Rankine and Cain-Bell types of formulas, the specimens in a number of cases were loaded with several hundred pounds of cast-iron proof weights to simulate soil pressure-heads of various amounts. The results of the shearing tests are plotted in Fig. 3. The tests show that there is an intrinsic or initial shearing resistance, q_0, for clays and loams, according to the contentions of Messrs. Cain* and Bell†, which, in general, will not occur noticeably with sands or freshly granulated earths, on account of the relative lack of cohesion.

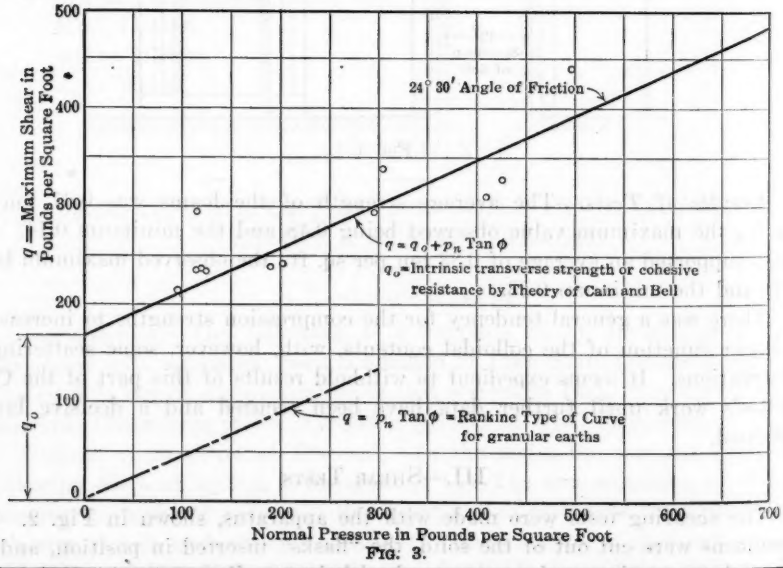


FIG. 3.

* *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1317.

† *Minutes of Proceedings*, Inst. C. E., Vol. 199 (1914–15), pp. 242–243.

IV.—Tensile Tests

A number of tensile tests were made on soils at depths varying from the surface to 8 or 9 ft. below, including loams, various yellow clays, and, finally, blue clays. The test-specimen holder was constructed somewhat on the principle of a clam-shell dredge bucket with sharp cutting-edges. The bucket was sunk slowly into the soil like a caisson by a gradual removal of the extraneous earth. The upward pull to failure was measured by a dynamometer. The colloidal contents, percentages of moisture, unit weights, etc., were determined on precisely sampled averages of four or five tensile strength specimens, taken immediately after the tests and sealed in glass jars. The results show (Figs. 4 and 5) that the tensile strength of a soil is dependent on the percentage of colloidal contents. The moisture content was approximately constant in the series of tests.

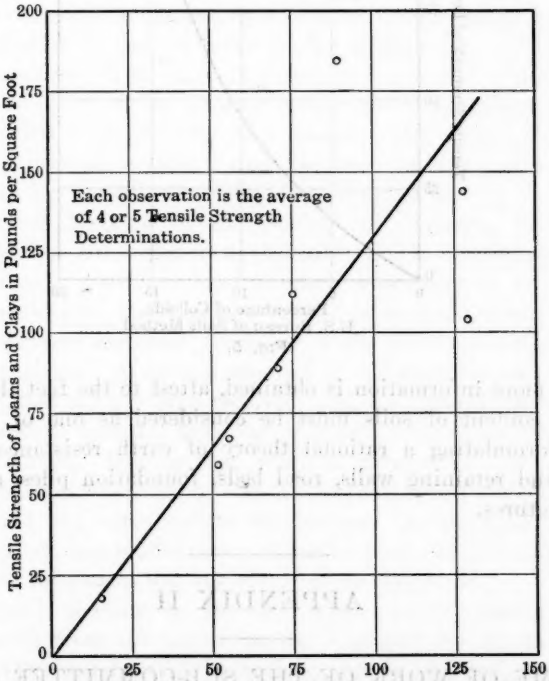


FIG. 4.

V.—Colloidal Determinations

A number of careful determinations of the colloidal contents of soils as they exist in Nature were made in addition to those cited. The tests were conducted in general conformity with the methods of the U. S. Bureau of Public Roads and U. S. Bureau of Soils. The data of the preceding figures, together with other important data which cannot be fully digested and inter-

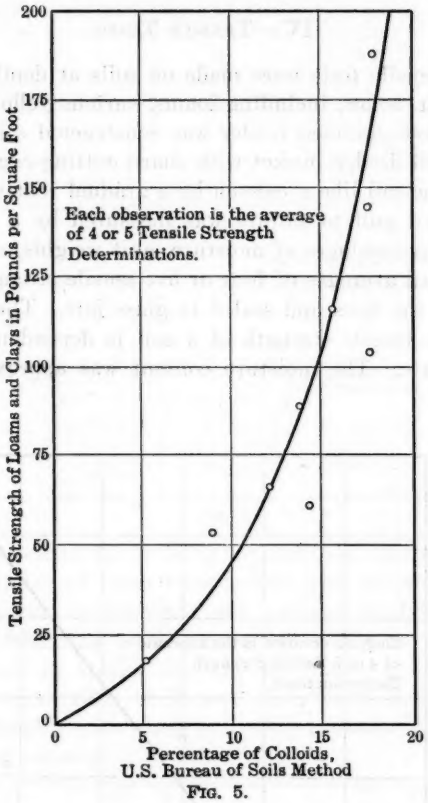


FIG. 5.

preted until more information is obtained, attest to the fact that the question of colloidal content of soils must be considered as one of the controlling factors in formulating a rational theory of earth resistance in connection with dams and retaining walls, road beds, foundation piles, and other engineering structures.

APPENDIX II

RESUME OF WORK OF THE SUB-COMMITTEE ON THE CODIFYING OF PRESENT PRACTICE

The scope of the work of this Committee is indicated by the resolution of the Board of Direction of January 7, 1913, the Committee having been appointed to "codify present practice on the bearing value of soils for foundations and report upon the physical characteristics of soils in their relation to engineering structures." Although they merge or overlap, these two subject-matters are separate and distinct, and have been so considered by the

Committee in its work ever since its appointment. In what follows, the first item delegated to the Committee for study will be considered in detail. At this time, it seems well to review and summarize the results obtained by the Committee on that part of its essential duties relating to codifying present practice.

During 1913, the work of this Committee consisted of organization, the laying down of a schedule of operations and subject-matters to be considered, and the adoption of means for collecting information. The general subject of codification was divided into six parts: (1) Tests; (2) the data from existing structures; (3) local practice; (4) local laws; (5) literature; and (6) other references. As a means of collecting information along these lines, circular letters, with categorical questions, were considered.

At the Annual Meeting of the Society in 1914, the Committee reported* progress, and stated that assurance of assistance had been received from the U. S. Bureau of Standards, and also from the U. S. Bureau of Mines, and that the preparation of a bibliography of the writings relating to soils was in progress. Regarding the second phase of the subject—that of the physics of soils—the Committee stated that it recognized that the classification and correlation of the soils of the United States was of the utmost importance, and must be its first consideration. The necessity of a correct classification of soils very early became apparent, and has been an outstanding feature in connection with the work of the Committee throughout.

At the Annual Meeting of the Society in January, 1915, the Committee reported† that a questionnaire (Circular No. 1) requesting information and data relating to tests and local practice, was about to be forwarded to members of the Society, and that the replies as received would be tabulated for record, study, and analysis. Samples of the soils reported on were also requested for analysis. A certain amount of information and a number of samples of soils had already been received, and, for various reasons, the Committee considered it essential again to emphasize the necessity of a correct classification of soils. During that year, in connection with classification, considerable attention was given to screen sizes, and a uniform screen specification was prepared and submitted to the Society. At the time of presenting the progress report, the bibliography had also been partly completed, and a list of about 400 references was submitted as an Appendix.

At the Annual Meeting in January, 1916, the Committee was able to report‡ the results from the questionnaire (Circular No. 1) which had been sent to nearly 8 000 members of the Society, and, in addition, the results from 1 000 personal letters which had been sent by the Committee to various engineers—whether members of the Society or not—whose positions or attainments seemed to indicate that they might have information of value to contribute. The results, both from the questionnaire and from the letters, were most disappointing; very few replies were received, the number being totally inadequate to justify any positive conclusions. It was also impracticable to

* *Proceedings*, Am. Soc. C. E., February, 1914, p. 104.
† *Proceedings*, Am. Soc. C. E., February, 1915, Papers and Discussions, p. 491.
‡ *Proceedings*, Am. Soc. C. E., March, 1916, p. 343.

classify the information received, because of the absence of uniform definitions for the various kinds of soils, and also because such tests as recorded were made in a variety of ways, often with devices lacking sensitiveness and open to other serious objections. Nevertheless, many of the replies contained interesting data of use to the Committee, some of which may prove to be of value in the conclusions.

The results of this work, under the head of "codification", tended still further to emphasize the necessity of a precise classification for soils, and much of the Committee's work during 1916 was devoted to the consideration of that subject, as a result of which a tentative method of classification was submitted. The Committee was also able to submit, as an Appendix, a subcommittee report of the U. S. Bureau of Standards, on earth experiments and apparatus. During that year the bibliography which had been submitted in 1915 was increased by the addition of about 500 additional references and re-submitted to the Society.

During 1916, further study was given to the matter of soil analysis and grading, and to the development of simple methods.* Because of the difficulty of comparing the results from tests submitted, owing to differences in methods of making them, the Committee also undertook the study of devices for making tests for the bearing value of soils and the development of a practical soil-testing apparatus. In the effort to obtain existing information, which might be tabulated and codified, the Committee continued its correspondence with other engineers—not only in the United States, but in various foreign countries—and about 1 500 additional letters were sent out. The results corresponded with those of the previous year, very little information of value for the purpose desired being obtained, and further work along that line was discontinued.

During 1917, although three meetings were held, the work and activities of the Committee were much restricted because of the World War, the members of the Committee, as individuals, devoting themselves to the service of the United States wherever possible.† The work of the Sub-Committee, including that of the U. S. Bureau of Standards, was also sub-ordinated to the work of the Government. The work of the Committee during the year was largely along three lines already in progress, namely, a further study of the granular analysis, grading, and charting of soils; a study of proposed definitions and classifications; and proposed devices for testing the bearing value of soils. The Committee also undertook a study of published data and the preparation of a descriptive classification of the manner in which soils give trouble.

The conditions existing in 1918 were the same as the previous year, and only matters already in hand were given attention. These included the development of the practical field device for measuring the actual supporting power of the soil, and considerable study was devoted to the subject.

* *Proceedings*, Am. Soc. C. E., August, 1917, Papers and Discussions, p. 1171.
† *Proceedings*, Am. Soc. C. E., February, 1918, p. 117.

At the Annual Meeting in January, 1920, the Committee submitted its report for 1919.* The report included, both in tabular form and as a diagram, the result of its investigation of building codes of the principal cities of the United States, showing the allowable load permitted to be placed on various types of soils. The types of soils listed, however, are those known by certain general names, and it does not follow that they have precisely the same characteristics in the different cities tabulated.

The Committee also submitted drawings for a proposed standard type of practical field-testing apparatus, for determining the bearing value of soil, the device having a limit of capacity of about 10 tons, and recommended its adoption by the members of the Society. The necessity of making such field tests by some means which would give reliable and concordant, and, therefore, comparable results, has often been brought out in the work of the Committee. As a result of studies and discussions by the Committee, it also submitted an analysis of the causes of failures of structures due to soils, discussing each cause of failure in some detail. The previous work on the question of grain size was also supplemented, and a revised schedule of grain sizes was submitted, together with recommendations as to methods of statement, plotting, and reporting the size. The Committee was also able to submit a revised Sub-Committee report of the U. S. Bureau of Standards, relating to the physical laws of soils under stress, including tests and the conclusions thereon.

During 1920, the Committee continued its activities along all the lines previously indicated.† It also took up several new subjects of interest, which had developed as a result of its work.

In work of this character, the lack of uniformity of ideas is surprising. The lack of uniformity in the definitions and classifications of soil is one of the difficulties which has confronted the Committee. As a result, a sub-committee undertook the preparation of a list of names or definitions which should be of more general application than those often used. The definitions of various soils, compiled from a number of dictionaries, were found, on comparison, to differ materially in the wording and sometimes in the sense, without differing in the essential facts. In different localities, the names for the same soil often differ greatly and the same name is sometimes applied to different soils.

In the list of definitions submitted by the Committee to the Society for consideration, it was attempted to provide a definition and name for each general type of material, which should be of broad application and yet sufficiently definite to identify the character of the material. This was done with the idea of stimulating discussion, and bringing out information regarding additional types or conditions of soils, particularly about some of the soils of the Far West with which the members of the Committee are not entirely familiar. However, it failed to produce the desired results, as was the case with the questionnaire.

In the report for 1920, the matter of soil classification was considered also from the mineralogical standpoint, and information as to the mineral composition and color characteristics of a number of surface soils was presented in

* *Proceedings*, Am. Soc. C. E., August, 1920, Papers and Discussions, p. 905.
† *Proceedings*, Am. Soc. C. E., February, 1921, Papers and Discussions, p. 9.

tabular form. The Committee also presented a simple method of obtaining and defining color composition, by a standardized method of blending different colors by the rotation of a sample of the soil with colored disks. Because of confusion in the use of the terms "settlement, compression, bearing power, allowable load," etc., the Committee presented a series of definitions of such terms as affect structures, for discussion or criticism.

The report for 1920 also included a résumé of the laboratory apparatus for soil testing, of which a number of special types had been developed by the Committee. As a result of this laboratory work and the development of the various devices, the influence of certain elements in soils in producing cohesion had been noted, and, in the report, attention was called to the marked cohesive properties of the colloidal element in clay and the fact that indications pointed to its being together with water, of extreme importance, if not the controlling factor, in the physics of soils. The Committee's table of grade sizes was re-submitted, therefore, revised to include the colloidal element which can be separated from the soil samples only by using a high-speed centrifuge.

In the report for 1921,* the Committee again reported the results from an attempt, by means of a questionnaire, to collect from the various members of the Society information on compression or settlement of soils, either from test loads or from existing structures. Through the office of the Secretary, more than 10 000 such circulars were issued, to which, during the year, only 65 members responded. Of that number, only 22 furnished information of the kind and in the form requested by the Committee. The questionnaire was, however, of a form from which copies could be readily made by blue-printing—the information to be transmitted on the copies and the original form retained indefinitely for similar use—and it was hoped additional information would be forwarded at a later date.

During 1922, relatively little work has been done along the line of codification of present practice. The Committee has endeavored to keep in touch with any work of testing for pressures or supporting power, and has been favored with a report of certain tests made by the Bronx Parkway Commission, in New York City. It was hoped that additional replies would be received to the questionnaire sent out in 1921, as this could readily have been done by reason of the character of the form furnished to the members of the Society, but, as additional information has not been made available to the Committee, it leads to one of several conclusions: That tests are not made to any great extent; that tests which are made, are not considered reliable by those making them; or that the members are not sufficiently interested in that phase of the bearing value of soils to furnish the information to the Committee.

The experience of the Committee, both with reference to the questionnaires and to correspondence with the various members of the Society, requesting records and observations in regard to bearing value of soils, has been most disappointing and leads to the conclusion that no considerable amount of definite information on the subject is available, and that it is useless to continue investigation along these lines. At almost every step in the work on

* *Proceedings*, Am. Soc. C. E., Papers and Discussions, p. 523.

codification, it has been apparent that no uniform basis existed, either of tests, classification, or methods, on which a code could be prepared; and the Committee's plan of carrying along with this work the subject of soil physics, and the laying down of a basis on which future observations and tests may be made, in a form to give comparable results, therefore, seems well justified. Only one valuable result may be said to have been achieved by the last questionnaire, relating to compression or settlement, and that was the fact that the practical soil-testing apparatus recommended by the Committee has been used at two localities, both on the Pacific Coast, with most satisfactory results.

In regard to codification, therefore, the Committee feels that these investigations have been carried along sufficiently to warrant the statement that information does not exist of a character, in a form, and of a degree of dependability, which will permit the Committee to prepare a code showing present practice in the bearing capacity of soils, as defined by local names and terms; and that, as a basis for preparing such a code at some future date, it is necessary to define the methods of classification—both for the soil as a whole and in its component parts—and both the apparatus used and the methods of making tests must be standardized, either in the forms suggested by the Committee, or modified as the Society sees fit. It is, therefore, both desirable and necessary that the members of the Society take a more active interest in the work and in the Committee, and that the discussion of the various features and proposals submitted be more widespread.

THE DESIGN OF EARTH DAMS

Discussion*

By Messrs. Thaddeus Merriman and John G. Hirsch

Thaddeus Merriman,† M. Am. Soc. C. E.—The speaker hopes that this paper will be thoroughly discussed. The value of any contribution to engineering literature cannot be fully appraised until after it has received extended consideration and study. The author presents the application of certain principles of underground flow to the design of earthen dams. These applications are carried to a seemingly logical conclusion, but the speaker believes that the argument presented is not entirely sound and should be used with caution until it has been more fully demonstrated.

Throughout the paper the author uses the term, "line of saturation", which he defines as "* * * the uppermost line of flow of the water through the dam and subsoil." From this definition, it is to be inferred that the theory presented is based on the supposition that below the line of saturation all the voids in the material are filled with water, and that, if a tube were to be driven below this line, the water level in the tube would coincide with it. Direct observations of this kind have been made in a number of structures, but the results obtained are not concordant, and the entire question as to water levels within earthen dams is in an embryonic state.

In 1918, two borings were made for the purpose of determining the condition of the materials in the embankments of the Ashokan Reservoir of the Catskill Water Supply System. These borings were located as shown in Figs. 50 and 51.

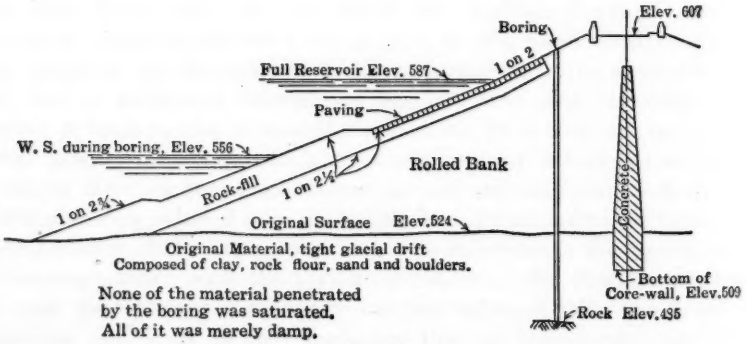


Fig. 50.

* Discussion on the paper by Joel D. Justin, M. Am. Soc. C. E., continued from September, 1923, *Proceedings*.

† Chf. Engr., Board of Water Supply, City of New York, New York, N. Y.

The boring shown in Fig. 50 was made in order to disclose the condition of the materials both in the embankment proper and in the original material beneath it. Dry samples were taken at 10-ft. intervals and their moisture content was determined. The results indicated an average moisture content of 12 to 18%, based on the dry weight. After drying, the evaporated water was replaced, and it was found that, in order to produce saturation, the addition of approximately 7% more moisture than that originally contained was required. Here is a case in which material lying up stream from a core-wall and from 0 to 70 ft. below water level was not saturated.

The material incorporated in this dam is the local glacial drift, composed largely of rock flour and clay with some fine sand. It is highly impervious to water and was placed in layers 4 in. thick after compacting with a 15-ton roller.

The boring shown in Fig. 51 was made in order to determine the condition of the embankment near the bottom, where the material had been placed by dumping into a pool of water, to Elevation 522, above which the embankment was rolled in 4-in. layers, as already described.

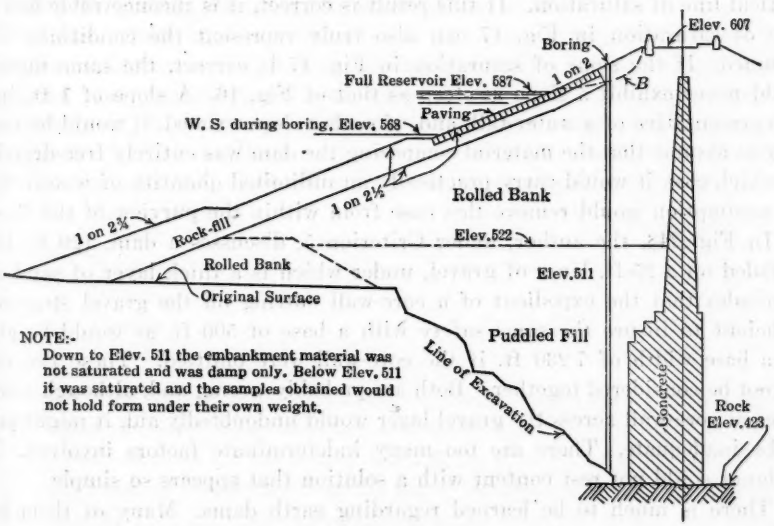


FIG. 51.

Between Elevations 597 and 522, the material which had been rolled showed the same moisture content as that described for Boring No. 1. Below Elevation 522, the material became softer, until at Elevation 511 it was saturated. Therefore, this material, which was placed in 1909, had not, in 9 years, given up its original moisture, whereas during the same length of time the material above it had received but little, if any, moisture, in addition to that which it held when placed.

This evidently is a case in which the theories proposed by the author would fail. In other words, this structure has no "line of saturation." It is a difficult and tedious operation to determine the line of saturation in a completed

embankment. The pipes driven are usually jetted or washed down, and the process results in injecting unknown quantities of water into the structure. Before the line of saturation can be determined, the effects resulting from this added water must be allowed to pass. Weeks, and possibly months, are required before trustworthy results can be obtained in a structure built of impervious material. No water-level results of value were obtained from the borings described.

In examining the results of the author's analyses, it is noted in Fig. 17*, that, with a 30-ft. dam built of fine sand on a foundation of the same material, a down-stream slope of 1 on 3½ is required for safety, whereas, in Fig. 18†, a dam of the same dimensions, but built of very impervious material, requires the same slopes to insure stability. The seepage in these two cases is widely different, and the speaker is of the opinion that, although the first is hardly to be classed as a dam, the second, if built, would never show a line of saturation even approximately similar to that indicated by the author.

In Fig. 16‡, a dam of fairly fine sand built on a foundation of coarse sand shows a leakage of 0.9 cu. ft. per min. per lin. ft. of dam and an essentially vertical line of saturation. If this result is correct, it is inconceivable how the line of saturation in Fig. 17 can also truly represent the conditions there assumed. If the slope of saturation in Fig. 17 is correct, the same material could never exhibit a slope as steep as that of Fig. 16. A slope of 1 ft. in 25 is representative of a water-fall, and, if such a slope existed, it would be necessary to assume that the material composing the dam was entirely free-draining, in which case it would carry practically an unlimited quantity of water. Such an assumption would remove this case from within the purview of the theory.

In Fig. 24§, the author, under Criterion 5, discusses a dam, 120 ft. high, founded on a 25-ft. layer of gravel, under which is a thick layer of sand, and concludes that the expedient of a core-wall cutting off the gravel stratum is sufficient to insure the same safety with a base of 500 ft. as would be given by a base width of 7 230 ft. if the core-wall were omitted. These two cases cannot be considered together. Both are probably wrong, and, although a small piece of core-wall across the gravel layer would undoubtedly aid, it might prove to be inadequate. There are too many indeterminate factors involved. The designer could not rest content with a solution that appears so simple.

There is much to be learned regarding earth dams. Many of them have failed, and it is incumbent on those responsible for their design to profit by whatever of knowledge or theory may be disclosed or developed. The speaker's criticisms of this paper are not to be considered as destructive; they aim only to show that the results obtained by the author's theory are not concordant at all points and that the arguments so concisely advanced should be applied with caution.

The basic error of the analysis with respect to the line of saturation is in the application and use of a formula for the flow of water on flat slopes through

* *Proceedings*, Am. Soc. C. E., May, 1923, p. 882.
† *Loc. cit.*, p. 883.
‡ *Loc. cit.*, p. 880.
§ *Loc. cit.*, p. 909.

materials in their natural positions, to the flow on steep slopes through materials in which the natural arrangement of the particles has been altered. Schlichter's formula undoubtedly gives good results, if conditions are reasonably comparable with those on which its constants were determined. This fact, however, does not warrant its extension to such widely differing conditions as those which obtain in earth dams.

Much investigation and study must be done before it will be possible to predict the position of the line of saturation with accuracy. This line is not only dependent on grain size, but also on the tension on the boundary surface between the water and the grain. In other words, the degree to which saturation occurs, depends on the extent to which the material will take on water by capillarity. Many materials when loosely placed will break down into soft mud, but when compacted will retain their form indefinitely. These same materials when placed with an excess of water will remain soft and not give up this excess for long periods. An extended investigation of these questions is greatly needed. A National hydraulic laboratory, as has been proposed,* would be the natural agency for research of this kind.

For the height of waves from crest to trough, the author presents the formulas of Stevenson, Hawksley, and Henny. It is interesting to compare the results of these expressions with an observed case of wave height on the Ashokan Reservoir, in December, 1915, the reservoir at that time being at full flow-line level. The length of reach or fetch was 4 miles, and the wind velocity was very high, probably 120 miles per hour. The comparison follows:

Formula.	Wave height, in feet.
Stevenson	4.00
Hawksley	3.90
Henny	8.50
Ashokan, observed	12 to 15

During this storm, large quantities of water were carried to a height of 12 ft. above the level water surface and washed out the paving, the rock-fill, and the embankment to the dotted line, *B*, in Fig. 51. At the same time, a heavy spray passed completely over the top of the dam, and the two stone guard walls, each 4½ ft. high. The quantity of water from this spray was sufficient to gully the down-stream embankment. A second storm in December, 1916, at the same place, and in spite of added protective paving, again did some damage. the waves being nearly as high as in 1915.

JOHN G. HIRSCH,† ASSOC. M. AM. SOC. C. E. (by letter).‡—In what might be termed the Middle Southwest, with particular reference to Missouri, Kansas, and Oklahoma, there are many small earth dams and some of moderate size. The formations at and near the surface are largely of fine sedimentary materials, clayey materials predominating. Many wells yield small quantities, or the yield diminishes with progressive clogging, the water in many

* *Proceedings*, Am. Soc. C. E., August, 1923, p. 1185.
† Prin. Asst. Engr., Benham Eng. Co., Kansas City, Mo.
‡ Received by the Secretary, August 24, 1923.

instances being unsatisfactory because of hardness and the high content of solubles.

The surface-water supplies in this district, although always silty and usually requiring purification, are being developed quite generally for public and industrial uses. The distribution of run-off is such as to require relatively large storage reservoirs in order to supply the demand during the drawing period from July to December, when the smaller streams are usually dry and the run-off from quite extensive drainage areas is practically nothing.

Many of these earth dams have been built of materials at hand and without engineering advice. Failures occur frequently, but are seldom reported in detail. Nearly every failure is caused by insufficient spillway capacity and consequent overtopping of the dam. It is probably a fact that spillway capacity requirements have been generally underestimated for drainage areas having limited retention characteristics, in view of the extensive flood-flow experience recorded in recent years. Little or no stream-flow data are available for the smaller streams in this territory, and estimates of maximum flows are of uncertain value because the floods generally spread over considerable areas, with the rivers "out of banks".

The writer's attention was directed recently to an occurrence at Holden, Mo., where an earth dam 15 ft. high was overtopped during a flood from a drainage area of about 5 sq. miles. This structure withstood successfully an overflow about 4 in. in depth for about 15 min. The dam is probably of clay throughout, the top and down-stream slopes being covered with a rank growth of vegetation and grasses. It is said that this dam has been overtopped on several occasions, but was largely destroyed by an extreme flood several years ago, and was rebuilt without sufficient spillway extension. The 108-ft. spillway is merely a concrete-paved earth section, with a down-stream slope of about 4 to 1, the crest being about 30 in. lower than the top of the dam.

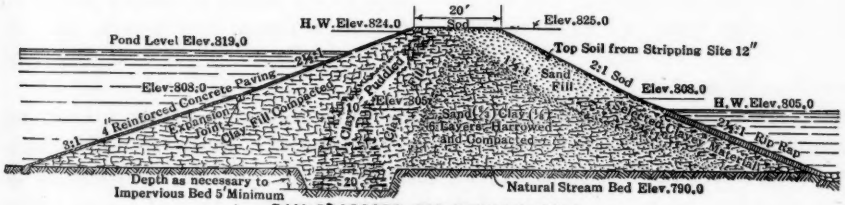
It is customary to build small dams with only a small freeboard allowance above the maximum flood level of the reservoir, because extreme floods on small drainage areas follow local storms, and the wind has usually abated or stopped by the time the maximum flow occurs over the spillway.

Although the clayey material used in building earth dams in this district, makes for water-tightness, the fine material is subject to erosion even with moderate water action.

The choice of materials for building an earth dam is usually restricted because there is little coarse or graded material; and most of the clay is of fine structure. Little gravel is to be found and nearly all the sand deposits are of fine material. Various treatises on earth dams either discourage or condemn the use of clay without a "body" of coarse or graded material.

With up-stream face materials that promote water-tightness, with the choice of materials restricted, and assuming that ample spillway capacity is provided, others of the criteria in the paper advance to first importance in the design of an earth dam which is to be reasonably safe, permanent, and economical, as well, both as to first cost and maintenance.

The writer and his associates have felt that the conditions in such cases merited special consideration, and some study has been given to various designs for the purpose of meeting local requirements in the best way. Fig. 52 is a tentative design of an earth dam section proposed in a report on an improved water supply for Cushing, Okla. The site selected is immediately below a natural basin on Big Creek, about 6 miles from the city. The dam, at the elevation to which it would now be constructed, would impound 400 000 000 gal. The tributary drainage area is approximately 31 sq. miles. The basin which would form the future reservoir is quite flat, and is bounded by hills. The stream channel winds through the basin with a normal slope of about 6 to 8 ft. per mile. At times of extreme flood the stream probably gets out of the channel and overflows the flats of the basin. The proposed dam consists of an earth section approximately 1 000 ft. long, and the concrete spillway section and approach channel are founded in the shale of the hill at one end.



DAM PROPOSED FOR CUSHING, OKLA.
MAXIMUM SECTION IN CREEK CHANNEL

FIG. 52.

Test pits and auger holes at various points along the axis of the proposed dam showed a uniform bed of slightly cemented fine brown sand, about 5 ft. deep, underlain by a thick stratum of red and black clays of very fine structure. The distribution of materials and the typical construction proposed, as shown by Fig. 52, were decided to be safe, economical, and properly adapted to the conditions at this site.

Reinforced concrete slab pavement on the up-stream slope retains the slope of the pure clay bank, prevents erosion and the drying out of the clay fill, and the consequent extensive cracking when the reservoir level is lowered during the long drawing period. "Gunite" facing has also been used successfully. It is believed that the puddled clay core will be properly supported by the fill and will retain perfect flexibility, with water-tightness, even if the expected local and general settlement occurs. It was thought that a masonry core-wall would offer no advantages and might prove to be hazardous if ruptured by settlement stresses. Tests showed that the fine sand overlying the site, although hard when dry, would not stand at any appreciable slope when placed in water, probably owing to capillary action and lubrication caused by silt. The down-stream slope would probably be stable if built of the sand and if not subjected to wetting other than by rain. The top and the down-stream slope are to be sodded to prevent erosion.

For the maximum section in the stream channel, the lower parts of both up-stream and down-stream slopes are flattened, as shown on Fig. 52. The

down-stream toe would be subject to saturation from back-water during floods. Somewhat pervious, selected, clayey material is proposed for the toe, with rip-rap protection, the object being to retain the toe material when saturated as well as to provide for drainage after back-water conditions or possible seepage from the reservoir.

The writer believes, with Mr. Justin, that the design of earth dams may well be based on analyses of the hydraulic and structural conditions to be expected. Broadly interpreted, and with a thorough understanding of the physical properties of the materials to be used, design in accordance with the criteria given should result in a safe and permanent structure.

The paper is an excellent and welcome contribution to earth dam engineering practice. The writer believes that similar information and data on good practice on other engineering subjects should be encouraged.

THEORETICAL FREQUENCY CURVES AND THEIR APPLICATION

Discussion*

By Messrs. L. Standish Hall and R. D. Goodrich.

L. Standish Hall,† Assoc. M. Am. Soc. C. E. (by letter).‡—The author deserves much credit for his work in adapting the theoretical frequency curves of Professor Karl Pearson for application to engineering problems. The writer believes, however, that he has not carried his work far enough. For any theory to be of practical application, it must cover all possible conditions. Professor Pearson's curves are of such a nature as to answer this requirement, but Mr. Foster has placed a limitation on his work by using only one of these curves.

As the writer has mentioned in former discussions of this subject, there are certain practical limitations to the application of theoretical methods to engineering statistics. Of these limitations, the principal one is the shortness of the records available. In many cases the constants, on which the selection of the proper type of frequency curve depends, cannot be determined with any degree of accuracy until records of considerable length are available. Theoretical frequency curves were developed originally for actuarial work, and the purpose for which they are used in such work is somewhat different from that for which it is sought to apply them to engineering problems. In actuarial work, the purpose is to interpolate data between limits rather definitely established, whereas, in engineering work, the purpose is to exterpolate short records beyond the known limits.

The difficulty in applying statistical methods to engineering data is illustrated clearly in a letter received by the writer from Professor G. Udny Yule shortly after the publication of the writer's paper "The Probable Variation in Yearly Run-Off as Determined from a Study of California Streams".§ Professor Yule states in part:

"The problem with which you are dealing is obviously a very difficult one, and the one comment that forces itself on my mind after a very hasty perusal of the paper concerns a point of logic rather than statistical method. Rainfall is very erratic: a good many observations, I believe, tend to show that various periodicities are involved in its changes from year to year. In these circumstances can any data that cover only some twenty or thirty years, however ingeniously combined and treated, give very certain guidance as to probable maxima and minima likely to occur only once in a much longer period such as a century?"

---

* Discussion on the paper by H. Alden Foster, Assoc. M. Am. Soc. C. E., continued from September, 1923, *Proceedings*.

† Asst. Engr., H. L. Haehl, San Francisco, Calif.

‡ Received by the Secretary, July 30, 1923.

§ *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 191.

From the actuarial standpoint, the results obtained from engineering data are only crude approximations; however, the application of these methods, although approximate, lead to a better understanding of the variations involved. For this purpose, a study of the theoretical curves is perhaps valuable. The author shows the development of the differential equation of the frequency curve* and then states: "In stream-flow studies, attention can be limited to Type III—limited range in one direction only, and skew—as it is obvious that run-off cannot have a value less than zero, and no definite upper limit can be assigned." This adoption of the Type III curve is not in accordance with the results obtained by the use of the criterion developed by Professor Pearson for the selection of the proper type of curve.

In order to show the use of the criterion, it will be necessary to explain the various types of frequency curves.† In order to determine the proper type of frequency curve to use, it is necessary to have the first, second, third, and fourth moments of the statistics about the mean, and to determine the corresponding moment coefficients, μ_1, μ_2, μ_3, and μ_4. From these, the coefficients, β_1 and β_2, are found as follows:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \text{ and } \beta_2 = \frac{\mu_4}{\mu_2^2}$$

All the moment co-efficients except μ_4 have been used by the author, and this higher moment was omitted by him, as it is not required for the solution of the Type III curve. β_1 is the square of the coefficient of skew, whereas β_2 is a coefficient which has not as yet been introduced in this discussion. With β_1 and β_2 determined, it is possible to decide on the proper curve to use to fit the data best, and to compute the value of all the coefficients in the equation of the curve.

The various types of frequency curves are all derived from the one differential equation, which has been given by the author, and is:

$$\frac{1}{y} \cdot \frac{d\,y}{d\,x} = \frac{a + x}{b_0 + b_1\,x + b_2\,x^2}$$

The various curves resulting from this equation depend on the roots of $b_0 + b_1x + b_2x^2$. By factoring this expression, it will be seen that the nature of the roots depends on the value of $\frac{b_1^2}{4\ b_0\ b_2}$, or by substituting values of β_1 and β_2, for b_0, b_1, and b_2, the criterion, K, is obtained:

$$K = \frac{\beta_1\,(\beta_2 + 3)^2}{4\,(4\,\beta_2 - 3\,\beta_1)\,(2\,\beta_2 - 3\,\beta_1 - 6)}$$

Quoting from Mr. Elderton's book:

"If this [the criterion] is negative the roots are real and of different sign (Type I), if positive and less than unity they are complex (Type IV), and if positive and greater than unity they are real and of the same sign (Type VI). This really covers all the cases, but just at the point where one type changes into another we can use a slightly simpler transition curve. Thus, when the criterion is $\propto$ one root is $\propto$ (Type III), when it is unity the

* *Proceedings*, Am. Soc. C. E., May, 1923, p. 830.

† A more complete description of the types of frequency curves will be found in "Frequency Curves and Correlation", by W. Palin Elderton, Chapter IV.

TABLE 9.—FREQUENCY CURVES.

No. of type.	Description of curve.	Equation to curve.*	Mode.	Criterion: K.	Criterion: β_1.	Criterion: β_2.
I.	Limited range in both directions (skew)	$y = y_0\left(1+\frac{x}{a_1}\right)^{v a_1}\left(1-\frac{x}{a_2}\right)^{v a_2}$	Origin	< 0	.....	.....
II.	Limited range in both directions (symmetrical)	$y = y_0\left(1-\frac{x^2}{a^2}\right)^{m}$	Origin	$= 0$	$= 0$	not $= 3$
III.	Limited range in one direction (skew)	$y = y_0\left(1+\frac{x}{a}\right)^{\gamma a} e^{-\gamma x}$	Origin	∞†	.....	.....
IV.	Unlimited range (skew)	$y = y_0\left(1+\frac{x^2}{a^2}\right)^{-m} e^{-v \tan^{-1}\frac{x}{a}}$	$-\frac{v a}{2 m}$	> 0 and < 1	.....	.....
V.	Limited range in one direction (skew)	$y = y_0 x^{-p} e^{-\frac{\gamma}{x}}$	$\frac{\gamma}{p}$	$= 1$	.....	.....
VI.	Limited range in one direction (skew)	$y = y_0 (x-a)^{q_2} x^{-q_1}$	$\frac{a q_1}{q_1 - q_2}$	> 1 and $< \infty$	.....	.....
VII.	Unlimited range (symmetrical)	$y = y_0 e^{-\frac{x^2}{2\sigma^2}}$	Origin	$= 0$	$= 0$	$= 3$

*The constants used do not have the same meaning in all equations.

†Strictly, when $2\beta_2 - 3\beta_1 - 6 = 0$; can also be used in cases of slight skewness, when the criterion is greater than 4.

two roots are equal (Type V), while when it is zero the roots are equal in magnitude but of opposite sign (Type II). The only other transition curve arises when $b_1 = b_2 = 0$, and the criterion is again zero (Normal Curve of Error, Type VII)."

The distance between the mean and the mode, or the value, a, in the differential equation is:

$$\frac{\sqrt{\mu_2}\ \sqrt{\beta_1}\ (\beta_2 + 3)}{2\ (5\ \beta_2 - 6\ \beta_1 - 9)}$$

As the skewness is the distance between the mean and the mode divided by the standard deviation,

$$\text{Skewness} = \frac{\sqrt{\beta_1}\ (\beta_2 + 3)}{2\ (5\ \beta_2 - 6\ \beta_1 - 9)}$$

The equations of the seven principal types of frequency curves is given in Table 9, together with their general characteristics.

In connection with the criterion, K, it may have any value from $-\infty$ to $+\infty$, and, from Fig. 12* of the discussion by Frederic H. Hapgood, Jun. Am. Soc. C. E., it will be noted how the types cover all possible values of the criterion and do not overlap.

This discussion has been given in order to show that the only rational method of selecting the proper type of frequency curve to fit any class of data is by means of the criterion. In order to find the proper type of curve to apply to stream-flow data, the writer has computed β_1, β_2, and K from the actual statistics used by him in preparing his Fig. 13.† (See Table 10.)

TABLE 10.—Frequency Curve Coefficients Computed from Stream-Flow Data.‡

c. v.	c. s.	Terms in series.	β_1	β_2	K
0.20	0.27	511	0.075	2.755	— 0.081
0.30	0.52	372	0.274	2.818	— 0.187
0.40	0.43	354	0.181	3.000	— 0.262
0.50	0.89	399	0.795	4.070	— 2.933
0.60	1.01	229	1.024	4.329	— 2.324
0.70	0.92	278	0.843	3.483	— 0.497
0.80	0.96	248	0.930	3.045	— 0.335
1.00	1.54	208	2.362	4.652	— 0.793
1.40	2.79	166	7.737	12.536	— 4.187

‡ The data from which the coefficients in this table were computed, and the method of obtaining the various series are explained fully in *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), pp. 238–244.

From Table 10, it will be noted that all these data, which cover practically the entire range of stream-flow statistics, can be fitted to the Type I curve, the only possible exception being the data having a coefficient of variation of 1.40, for which the Type III curve could apply, as K is greater than 4.

At this point, the writer wishes to correct an erroneous impression given by Mr. Foster, to the effect that the duration curves shown in Fig. 13† of the writer's paper previously mentioned, follow the Type III distribution, as

* *Proceedings*, Am. Soc. C. E., September, 1923, p. 1645.

† *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 244.

indicated in the author's Fig. 5.* Mr. Foster states: "The percentage of time at which one of these curves crosses the mean is an index of the *c. s.* which will apply to this curve, and the proper value of *c. s.* may be determined from the percentages given in the second column of Table 2 in this paper."

The method used by the author merely gives the *c. s.* for each curve assuming that the data followed the Type III distribution—hence, the close agreement with the requirements of the Type III curve. The percentage of terms below the mean for the series given in Table 10 has been previously given,† and a calculation of the *c. s.* by the author's method from these percentages will show wide variations from the values of *c. s.* computed in Table 10.

In order to illustrate better the effect of β_1 and β_2 on the selection of the type of frequency curve, Fig. 13 was prepared. By considering the equation for K, it will be noted that two conditions will give a value of infinity; these are, $4\,\beta_2 - 3\,\beta_1 = 0$ and $2\,\beta_2 - 3\,\beta_1 - 6 = 0$, the latter being the criterion for the Type III curve. These two lines represent the extreme limits for the Type I curve. There are two other limits applicable to the Type I curve, namely, when the distance between the mean and the mode becomes infinite, or when $5\,\beta_2 - 6\,\beta_1 - 9 = 0$, and when the constant, r, in the Type I equation equals zero, or when $\beta_2 - \beta_1 - 1 = 0$.‡ It will be noted that the lines representing all these equations intersect at $\beta_1 = -4$, $\beta_2 = -3$.

As the limits of the various types depend on K, values of β_1 and β_2 for values of K equal to $+1$, -1, $+4$, and -4, were computed. This was done by transposing the equation for K into the following quadratic:

$$4\,\beta_2\,(2\,\beta_2 - 6) - \left[3\,(2\,\beta_2 - 6) + 12\,\beta_2 + \frac{(\beta_2 + 3)^2}{4\,K}\right]\beta_1 + 9\,\beta_1^2 = 0$$

By assuming values for β_2 and K, the solution could be made for β_1. The resulting curves are plotted on Fig. 13, and the limits of the various types of frequency curves shown. When considering stream-flow data, the curves representing Types IV, V, and VI can be rejected at once, because one condition necessary for the use of these curves is that the mode should be above the mean, a condition which does normally exist in stream-flow data. The types of curves usually to be expected are Types I and III, or, in certain cases, Types II and VII. The great bulk of such data would undoubtedly fall in the Type I group.

As has been stated, values of β_1 and β_2 satisfying the criterion for the Type I curve lie between the limits, $5\,\beta_2 - 6\,\beta_1 - 9 = 0$ and $2\,\beta_2 - 3\,\beta_1 - 6 = 0$. The number of frequency curves that could be drawn to satisfy these conditions are practically infinite. The great difficulty in applying the Type I curve to stream-flow data is in the calculation of a reliable value of β_2. It has been shown§ that a reliable value of *c. s.*, or β_1, cannot be obtained until a record of some length is available, and in the computation of β_2, this

* *Proceedings*, Am. Soc. C. E., May 1923, p. 849.

† *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 239.

‡ The values of β_1 and β_2 lying between these two limits, result in many special types of curves, see "Frequency Curves and Correlation", by W. Palin Elderton, Addendum.

§ *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), pp. 245–247.

difficulty is greatly magnified. In fact, any values of β_2 computed from the records of individual streams that are at present available, would be practically worthless. The only solution apparently is to determine β_2 in terms of β_1 so as to represent average Type I conditions. Such average conditions are expressed in the equation, $3\beta_2 - 4\beta_1 - 7 = 0$, which gives the average value of β_1 for values of β_2 between the limits for the normal Type I curves.

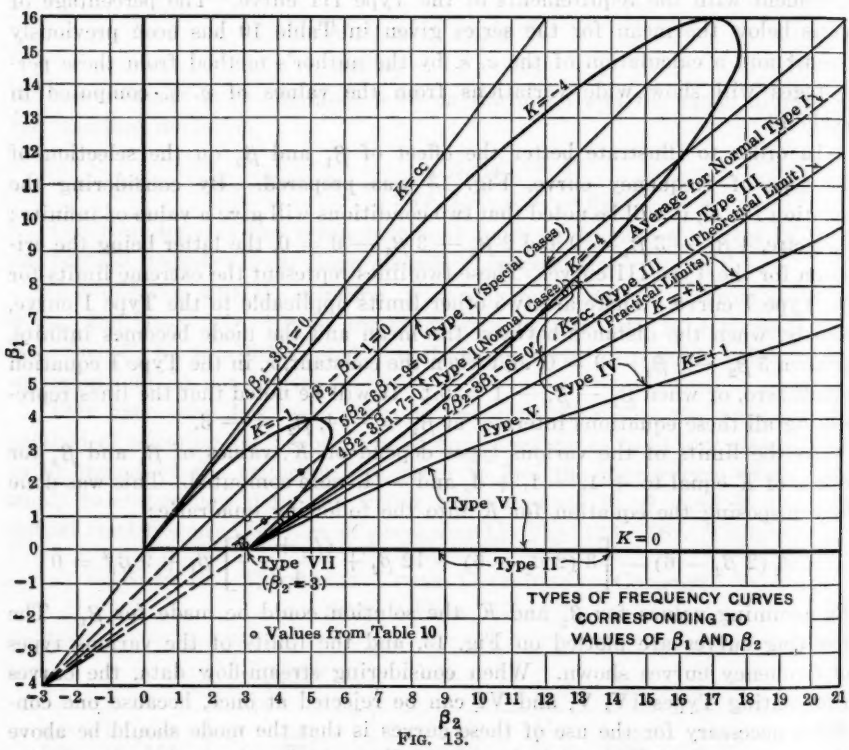


FIG. 13.

On Fig. 13 has been plotted the values of β_1 and β_2 given in Table 10. The average values of β_1 and β_2 for coefficients of variation of 0.20 to 1.00, inclusive, are 0.810 and 3.519, respectively, and it will be seen that this point lies practically on the line selected to represent average conditions.

The Type I curve, which the writer selects as best representing stream-flow data, is described in Table 9 as being skew and having a limited range in both directions. The fact that the range of the curve is limited in both directions should not be considered any objection to the use of the Type I curve, because, if a curve having an unlimited range is used, the area above a certain point is neglected as being insignificant. This has been noted by the author.* The equation of the Type I curve is:

$$y = y_0 \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2}, \text{ where } \frac{m_1}{a_1} = \frac{m_2}{a_2}$$

* *Proceedings*, Am. Soc. C. E., May, 1923, p. 847.

The constants in this equation are calculated in the following order:

$$r = \frac{6\,(\beta_2 - \beta_1 - 1)}{6 + 3\,\beta_1 - 2\,\beta_2}$$

$$b = \frac{1}{2}\,\sqrt{\mu_2}\,\sqrt{(\beta_1\,(r+2)^2\,16\,(r+1))}$$

m_2 and m_1 are given by

$$\frac{1}{2}\left[r - 2 \pm r\,(r+2)\,\sqrt{\frac{\beta_1}{\beta_1\,(r+2)^2 + 16\,(r+1)}}\right]$$

and $a_1 + a_2 = b$, and $a_1 \div m_1 = a_2 \div m_2$

$$y_0 = \frac{N\,m_1^{\,m_1}\,m_2^{\,m_2}\,\Gamma\,(m_1 + m_2 + 2)}{b\,(m_1 + m_2)^{m_1 + m_2}\,\Gamma\,(m_1 + 1)\,\Gamma\,(m_2 + 1)}$$

$$\text{Skewness} = \frac{1}{2}\,\sqrt{\beta_1}\,\frac{r+2}{r-2}$$

$$\text{Mode} = \text{Mean} - \frac{1\,\mu_3\,(r+2)}{2\,\mu_2\,(r-2)}$$

By substituting $3\,\beta_2 = 4\,\beta_1 + 7$ in the equations just given, the constants may be reduced to the following expressions:

$$r = 6$$

$$b = 2\,\sqrt{\mu_2}\,\sqrt{4\,\beta_1 + 7} = 2\ c.\ v.\ \sqrt{4\ (c.\ s\ )^2 + 7}$$

$$m_2 \text{ and } m_1 = 2 \pm 6\,\sqrt{\frac{\beta_1}{4\,\beta_1 + 7}} = 2 \pm \frac{6\ c.\ s.}{\sqrt{4\ (c.\ s.)^2 + 7}}$$

$$a_1 = \sqrt{\mu_2}\,(\sqrt{4\,\beta_1 + 7} - 3\,\sqrt{\beta_1}) = c.\ v.\ (\sqrt{4\ (c.\ s.)^2 + 7} - 3\ c.\ s.)$$

$$a_2 = \sqrt{\mu_2}\,(\sqrt{4\,\beta_1 + 7} + 3\,\sqrt{\beta_1}) = c.\ v.\ (\sqrt{4\ (c.\ s.)^2 + 7} + 3\ c.\ s.)$$

$$\text{Skewness} = \sqrt{\beta_1} = c.\ s.$$

$$\text{Mode} = \text{Mean} - \frac{\mu_3}{\mu_2} = \text{Mean} - (c.\ v.)\ (c.\ s.)$$

By using these simplified constants, the integration and solution of the Type I equation can be accomplished in a manner similar to that used by the author in solving the Type III equation. Lack of time prevents the writer from making this solution.

The constant, a_1, represents the distance from the beginning of the curve to the mode, whereas the constant, a_2, represents the distance from the mode to the upper limit of the curve. Using d as the distance between the mean and the mode, the distance from the beginning of the curve to the mean is:

$$a_1 + d = c.\ v.\ (\sqrt{4\ (c.\ s.)^2 + 7} - 2\ c.\ s.)$$

As $a_1 + d$ cannot have a value greater than unity, the maximum value of $c.\ v.$ for any value of $c.\ s.$ is:

$$\text{maximum } c.\ v. = \frac{1}{7}\left(\sqrt{4\ (c.\ s.)^2 + 7} + 2\ c.\ s.\right)$$

In Fig. 14, the equation of the curve just mentioned has been plotted, together with the similar equation for the Type III curve, $c.\ v. = \frac{1}{2}\ c.\ s.$

There is also plotted on Fig. 14, the values of *c. v.* and *c. s.* from Table 10, and the position of these points shows that these data (with the exception of one value) belong in the Type I class. As indicated on Fig. 14, if *c. s.* lies between 0 and 1.0, the data are almost certain to belong in the Type I class; and if *c. s.* lies between 1.0 and 2.4, the data will probably belong in the same class; but if *c. s.* is greater than 2.4, the chances of the data falling in the Type III class become greater, especially if *c. v.* is less than 1.2. These conclusions were derived from Fig. 13, and were based on a theoretical consideration of the probability of K being less than or greater than -4 between the limits of the normal Type I on Type III curves.

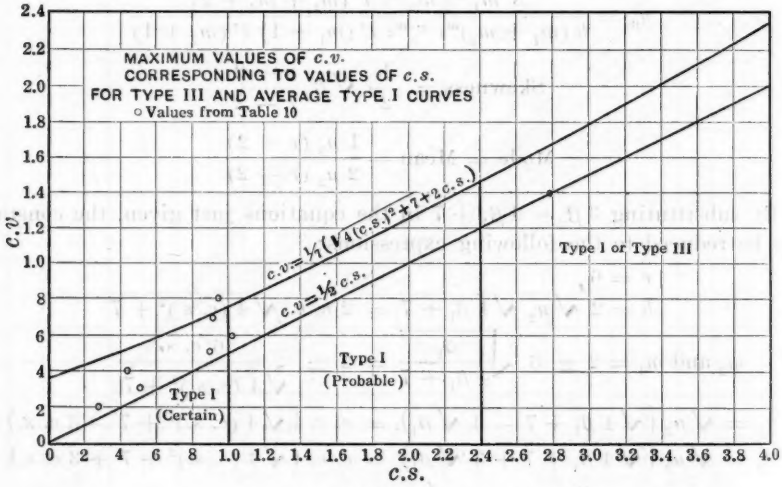


FIG. 14.

The writer's discussion, as to the best type of frequency curve to apply to stream-flow statistics, has been limited thus far to the results obtained from the series derived from the combination of several streams having approximately the same coefficient of variation. As has been mentioned previously, it is impractical to make a study involving the calculation of β_2 by any other method with the records available. The only simple criterion to be applied to the short records existing is the one just discussed; that is, whether the application of the Type I or Type III curve will keep the start of the curve at or above zero run-off. There were two summaries of run-off data presented in the writer's paper, one by Allen Hazen, M. Am. Soc. C. E.* and the other by the writer.† Mr. Foster states in his conclusion that "the main limitation to be placed on the use of these curves (Type III) is the fact that the coefficient of skew must be greater than twice the coefficient of variation". Using this criterion, there are only 14 streams out of 50 in Mr. Hazen's Table 4 to which these curves would apply; and only 11 streams out

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), Table 4, p. 217.
† *Loc. cit.*, Table 17, p. 248.

of 40 in the writer's Table 17 to which they would apply. Using the writer's criterion for the position of the starting point of the frequency curve, there are 38 streams in Mr. Hazen's Table 4, and 32 streams in the writer's Table 17 to which the average Type I curve would apply. As a matter of fact, considering the relation between the coefficient of variation and the coefficient of skew, there are only two streams in the two tables, (the Kiewa River and the Sweetwater River) that would not best be fitted by the Type I curve, excepting, however, a few streams that apparently have a negative *c. s.*

As was stated in the beginning of this discussion, Mr. Foster's work is noteworthy, but it was not carried far enough. If skew curve factors for both the Type I and Type III curves were available, engineers would have a much better basis for projecting engineering statistics than at present. There are some classes of data which, to a great extent, would fall in the Type III class, but it is the writer's opinion that the greater part of the data belong in the Type I class. A more complete study of data other than yearly stream flow, for example, rainfall or flood flows, would perhaps give more information on the subject.

R. D. GOODRICH,* M. AM. SOC. C. E. (by letter).†—This paper should be of great assistance in the analysis of hydrological and other data by the use of frequency curves. With the more intensive study now given the economic features of many problems in hydraulic engineering, the service rendered by Mr. Foster in making clear the theory of skew frequency curves and their practical application in engineering work, will be greatly appreciated by many engineers.

For several years, the writer has made use of graphical methods in the study of precipitation and run-off records, usually with the aid of probability paper, but the ever-present skew in the resulting curves led recently to the use of an empirical formula. The equation selected was the author's Equation (8),‡ but with three constants instead of two. In the same nomenclature as that of the paper, this equation is:

$$y = c\, z^{p'}\, e^{-nz}, \text{ or } y = c\, z^{p'}\, b^{z}$$

A theoretical discussion of this last equation with a method for the graphical determination of the values of the constants may be found in Running's "Empirical Formulas." It may be said briefly that if the data are such that they can be arranged as a frequency distribution with the values of z (rainfall or run-off) in an arithmetical series, and that if first differences of the logarithms of z, when plotted against the first differences of the logarithms of y (frequencies), fall on or near a straight line, then this equation will approximately represent the relation of the data. The slope of this line gives the value of the exponent of z, whereas n (or b) can be determined from the intercept. The constant, c, can then be computed from the average of the values computed from the known data, or so as to fit a controlling point

* Engr. in Chg. of Works, Chihli River Comm., Tientsin, China.
† Received by the Secretary, August 1, 1923.
‡ *Proceedings*, Am. Soc. C. E., May, 1923, p. 834.

as the maximum point on the curve.* When $n = 1$, this expression becomes identical with the author's Equation (8) and p' becomes equal to p. Differentiation and examination of the equation containing three constants, show that it fulfills all the requirements set down on page 830† of the paper as necessary for the general formula for frequency curves. Equation (8) is often a more convenient form than Equation (3),‡ as the original data can then be used in many cases, without conversion into terms of the mean. In Equation (8), the values of the two constants depend on the co-ordinates of the maximum point of the curve. If the data are very numerous, as in long-time records, these co-ordinates can often be determined with sufficient accuracy by inspection, and the values of the constants can then be determined by the formulas on page 834† of the paper.

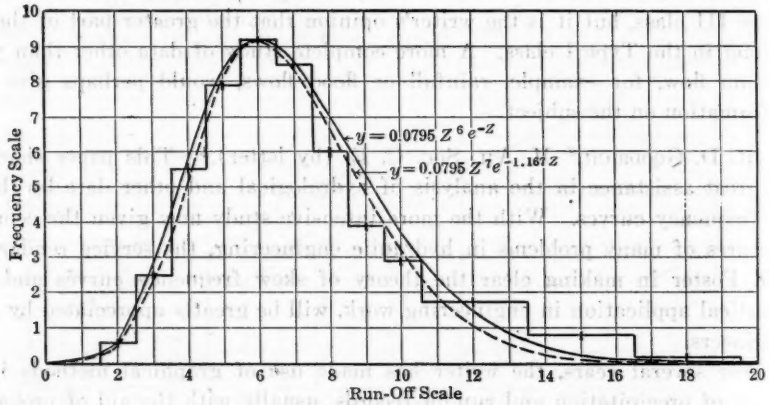


FIG. 15.

To show the possibilities of the use of the third constant in the equation mentioned previously, an assumed frequency distribution was drawn at random and adjusted only enough to illustrate the desired points. This distribution is shown in Fig. 15. The maximum point on the curve was taken at the center of the highest step and the constants were then computed, giving $y = 0.0795z^6e^{-z}$. This equation gives the curve shown in Fig. 15 by the solid line, which is about as close an approximation to the assumed frequency distribution as could be expected.

---

* $y = c\,z^p\,e^{-nz}$

$\log y_k = \log c + p \log z_k - n z_k$

$\log y_{k+1} = \log c + p \log z_{k+1} - n z_{k+1}$

Subtract the first equation from the second, then:

$$\log y_{k+1} - \log y_k = p\,(\log z_{k+1} - \log z_k) - n\,(z_{k+1} - z_k)$$

or,

$$(\Delta \log y) = p\,(\Delta \log z) - n\,(\Delta z)$$

where Δ indicates the first difference.

In an arithmetical series, Δz is the common difference and is, therefore, a known constant in the equation given previously, which is in the form of the slope equation of a straight line. Running's method is to determine the constant, c, first, but the writer prefers the method given previously for frequency curves. When common logarithms are used, the last term becomes $-n\,(\Delta z)\log_{10} e$.

† *Proceedings*, Am. Soc. C. E., May, 1923.

‡ *Loc cit.*, p. 833.

With the maximum point fixed, it may be desirable to make the hump either broader or narrower, that is, to cause the curve to fall outside the solid curve or inside like the dotted curve. This can be done by means of the third constant, n, by using the relation, $p' = np$. If p' is greater than p, n will be greater than 1 (unity), and the new curve will fall inside the first curve and *vice versa*. The curves will be tangent to the X-axis at the ends and to each other at the maximum point, as shown. The writer has not determined just how this third constant could be included in the theoretical discussion of skew frequency curves. It might involve a "fourth moment". At any rate, it seems to be beyond the writer's ability to make the investigation.

It should be noted that the equation with three constants will probably only prove useful when the data are sufficient to make a reasonable frequency distribution, as with long-time records of rainfall or monthly stream discharges. For short-time records, the method based on Table 2,* as presented in the author's applications to engineering work, and detailed under the "Numerical Example," † is by far the easiest and most practical. This method, with the help of the tables in the paper, should greatly increase the scientific investigation of available data in connection with many engineering projects, and the author is to be commended for his clear and able presentation of this subject.

* *Proceedings*, Am. Soc. C. E., May, 1923, p. 844.

† *Loc. cit.*, p. 851.

THE DISINTEGRATION OF CEMENT IN SEA WATER

Discussion*

By Messrs. B. Jeanneret, A. H. Rhett, I. Hiroi, and Theodore Belzner.

B. Jeanneret,† Esq. (by letter).‡—Additional information on the tests of *ciment fondu* will be found in the following discussion: The original chemical composition of cements is no guaranty of their value. It is the chemical combination which takes place in the moist mortar which is the principal factor in their ability to oppose the destructive action of sea water.

The combined silicates and not the mixtures of silicates produced after burning, are the agents which furnish the cohesive strength. It is for this reason that pure cements (Portland, natural cements, well burned hydraulic lime) offer a much better guaranty of the value of a mortar than the mixed cements (mixed cements or insufficiently burned lime rich in free lime). The value of adding trass, volcanic ash, etc., lies in the chemical re-action which they are able to cause.

The more prompt the chemical re-action of the silicates to hydration, the more readily the gelatinization takes place and the consequent impermeability of the mortar.

Porosity is the second important factor governing the value of mortars and concrete in water, that is, the physical resistance opposed to the penetration of water and, consequently, to the transmission through the pores of a saturated solution composed of the soluble salts of the mortar cement. This second kind of impermeability can be obtained by the selection of sands, fine pulverization, etc. The work of the engineers has considerably bettered these physical values of mortar and concrete.

The writer has made some tests of the impermeability of cements and other hydraulic binding agents, following the methods indicated by Le Chatelier, and the results are given in Tables 3 and 4.

The impermeability may be tested by the penetration of test samples by a solution giving colored re-actions. The limit of the coloration, which can be easily determined, gives an indication of the porosity of the mortar or cement. Tests were made on samples having the form of a figure "8" and on cylinders 30 mm. in diameter.

As re-agents, Le Chatelier suggests alkaline sulphurs which produce a black coloration and are without any chemical influence on the cement. Better results were obtained by using a 10% solution of calcium ferro-cyanide (K_4 Fe $(CN)_6$).

---

* Discussion on the paper by William G. Atwood and A. A. Johnson, Members, Am. Soc. C. E., continued from September, 1923, *Proceedings*.

† Ingénieur-Chimiste, Liège, Belgium.

‡ Received by the Secretary, July 9, 1923.

TABLE 3.—ANALYSIS AND COMPRESSIVE STRENGTH OF THE CEMENTS USED, AND TENSILE STRENGTH OF 1 : 3 RAMMED MORTAR IN TEST D.

Kind of cement.	Analysis of Cement.						Compressive strength of neat cement, in kilogrammes per square centimeter.	Tensile strength of 1:3 rammed mortar, in kilogrammes per square centimeter.
	SiO_2.	Al.	Fe.	Ca O.	Mg.	SO_3.		
1 Portland	21	7	3	62	1.7	1.3	350	32.5
2 Natural	19	8	4	56	2.0	1.6	270	25.7
3 Hydraulic lime	18	7	4	55	1.7	1.8	200	18.4
4 Hydr. lime, inferior quality.	16	3	2	59	2.0	2.5	150	....
5 *Ciment fondu*	15	40	10	32	1.0	1.1	800	62.0
6 Lime	13	3	2	59	2.0	1.9	135	....

All these tests give the strength at 28 days, and are made after the Swiss standard, with 1:3 mortar and calibrated standard sand.

TABLE 4.—PERMEABILITY OF CEMENTS.

No.	Experiment.		Kind of cement.	Permeability, in millimeters.
A	30-mm. cylinders of neat cement. Remained 3 months in the open air after fabrication. Plastic paste after Vicat's test.	1	Portland	1
		2	Natural	1-2
		3	Hydraulic lime	7-8
		4	Hydr. lime of inferior quality	8-13
		5	*Ciment fondu*	None
		6	Lime	12-15
B*	Mortar 1:3, plastic; moulds filled by hand without compression; river sand, screened; passed 335-mesh sieve, No. 50; retained on 484-mesh sieve, No. 60. Tested 28 days after fabrication. (*a*) Hardened in moist air.	1	Portland	1-2
		2	Natural	1-3
		3	Hydraulic lime	4-5
		4	Hydr. lime, inferior quality	7-8
		5	*Ciment fondu*	0.5
		6	Lime	10-15†
B	(*b*) The same mortar, remained in water after setting. Tested 28 days after fabrication.	1	Portland	1
		2	Natural	1-2
		3	Hydraulic lime	4-5
		5	*Ciment fondu*	None‡
C	Rammed mortar, 1:3 for standard test. Test of half-samples of 8 form. Remained 1 year in the open air after the tensile test.	1	Portland	2-3
		2	Natural	2-3
		3	Hydraulic lime	6-7
		6	Lime	10-11
		5	*Ciment fondu*	2-3§
D	Rammed mortar, 1:3 for standard test. Tested 28 days after fabrication. Test samples removed from water.	1	Portland	3-4
		2	Natural	3-4
		3	Hydraulic lime	5-7
		5	*Ciment fondu*	0.5-2‖

* The samples remained in the coloring solution from 24 to 48 hours.

† Complete penetration in cylinders.

‡ No penetration.

§ Electric or Alca cement at the age of 3 months.

‖ The dark coloring of the cast cement makes it difficult to observe exactly the depth of penetration of the red color.

In order to show the impermeability, the specimens must be immersed for a certain time in the solution mentioned. After cleansing with water, the samples are broken and colored by a cupric sulphate solution ($Cu\ SO_4$). This produces a red coloration of the salts which have formed in the pores of the specimens. This coloration persists for a long time, thus making easy the observation

of the limit of penetration and comparison with other specimens. From these tests it seems to be certain that the resistance of the hydraulic binding agents to permeability is not directly proportional to their tensile and compressive strengths, and that the chemical constitution of the silicates has an important bearing on the impermeability of mortar.

A. H. RHETT,* Assoc. M. Am. Soc. C. E. (by letter).†—This paper has given some interesting information on a subject of practical importance. A better explanation might be advanced for the rapid disintegration of a porous concrete than that given, which states: "Any increase in permeability increases the area exposed to salt water and, consequently, the rapidity of disintegration."

Most of the tests or observations have shown that very little, if any, disintegration occurs where the concrete is constantly submerged, but that alternate wetting and drying are essential conditions, and that even then it does not always occur, or occurs to varying degrees, although the same cement might have been used.

This suggests the rational explanation that the salt water in its normal condition is not sufficiently strong in sulphates to cause disintegration. If, however, a porous surface is present, and it is alternately wetted and dried, each flushing of the surface will fill the pores and each drying will leave a small deposit of the salt in them. A constant repetition of this process would cause concentration in the pores, which would hasten the chemical action, and, if saturation occurred, this would be accompanied by crystallization that would tend to disrupt the concrete.

I. HIROI,‡ M. Am. Soc. C. E. (by letter).§—To those who may have taken the trouble to read the writer's paper entitled "On Long-Time Tests of Portland Cement,"‖ presented to the Society in 1913, it may be of interest to know of the subsequent results of the tests which then covered only 14 years. The three diagrams showing the time-strength curves of briquettes kept in sea water, air, and fresh water then given are brought up to date in Figs. 3, 4, and 5. They present no remarkable features, beyond showing that briquettes kept in sea water continue to be less in strength than those placed in air or fresh water. The tensile strength of neat cement briquettes, which fell rapidly in the first 2 years have neither recovered their former strength nor suffered much additional loss. That such low tensile strength of neat cement briquettes placed in sea water is due to excessive crystallization appears to be probable, in the light of another noteworthy fact that, in almost all cases, their compressive strength stands higher than that of sand-mixed briquettes, varying from 4 500 to 8 000 lb. per sq. in.

The results of such laboratory tests serve merely to show, to a certain extent, the chemical action of sea water on cement. The most trying condi-

* Summit, N. J.
† Received by the Secretary, August 13, 1923.
‡ Tokyo, Japan.
§ Received by the Secretary, August 13, 1923.
‖ *Transactions*, Am. Soc. C. E., Vol. LXXVI (1913), p. 1027.

tions for materials used in sea water are the alternate wetting and drying, and exposure to waves and currents, all of which help to accelerate the chemical action, besides exercising physical actions proper to them. Unless

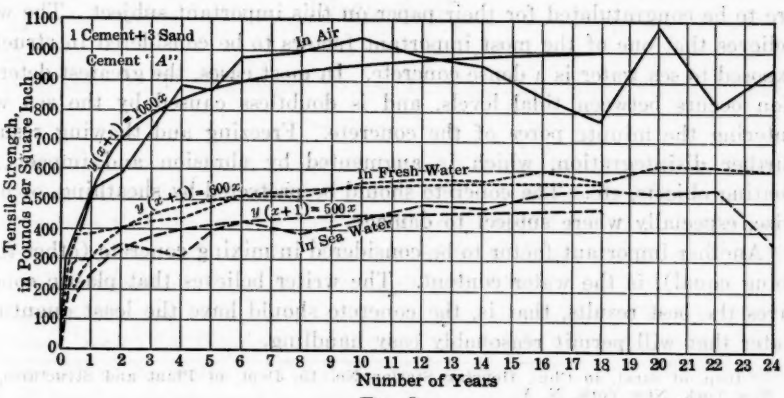


FIG. 3.

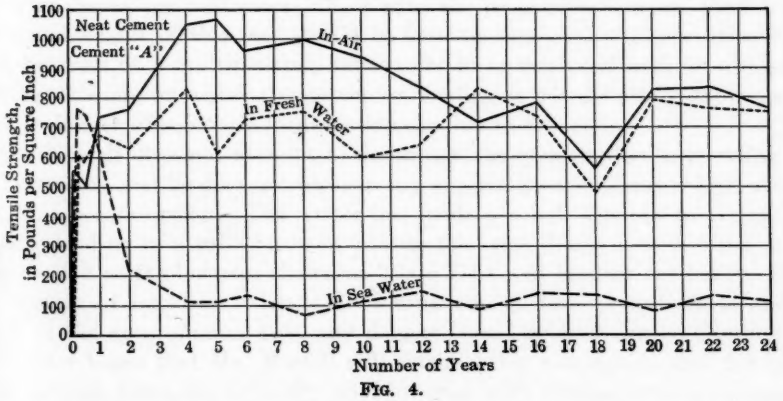


FIG. 4.

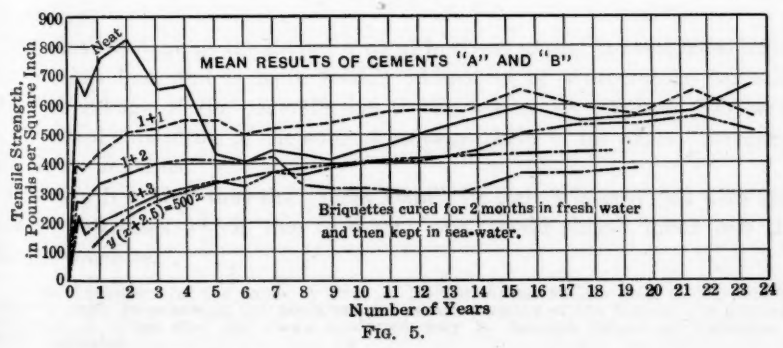

MEAN RESULTS OF CEMENTS "A" AND "B"
Neat
1+1
1+2
1+3
$y(x+2.6) = 500x$
Briquettes cured for 2 months in fresh water and then kept in sea-water.
Tensile Strength, in Pounds per Square Inch
Number of Years

FIG. 5.

these conditions are reproduced in the tests, the real action of sea water could not be expected to appear fully in the results. For this reason, it is hoped that systematic tests will be conducted under conditions approximating those

in which cement structures are to stand, and that the subject will be most thoroughly investigated.

THEODORE BELZNER,* AFFILIATE, AM. SOC. C. E. (by letter).†—The authors are to be congratulated for their paper on this important subject. The writer believes that one of the most important factors to be considered in structures exposed to sea water is a dense concrete. In most cases, the greatest deterioration occurs between tidal levels, and is doubtless caused by the sea water entering the minute pores of the concrete. Freezing and thawing result in further disintegration, which is augmented by abrasion and impact from floating objects, etc. The concrete should be protected by sheathing, or otherwise, especially where subject to damage.

Another important factor to be considered in mixing concrete (other things being equal), is the water content. The writer believes that plastic concrete gives the best results, that is, the concrete should have the least quantity of water that will permit reasonably easy handling.

* Insp. of Steel, in Chg., Housing Station No. 15, Dept. of Plant and Structures, City of New York, New York, N. Y.

† Received by the Secretary, August 20, 1923.

IMPROVED TYPE OF MULTIPLE-ARCH DAMS

Discussion*

By Messrs. B. F. Jakobsen, Victor H. Cochrane, and William Cain.

B. F. Jakobsen,† M. Am. Soc. C. E. (by letter.)‡—The author deserves praise for attempting to extend the field of the multiple-arch dam; and the writer agrees that the gravity dam is not deserving of the great confidence it generally inspires, and that its factor of safety is always much less than that of a well-designed multiple-arch or single-arch dam.

The author assumes *a priori* that it has been the danger of buckling of the buttresses, which in the past has limited the height of multiple-arch dams. On page 1068,§ he states: "For dams higher than 125 ft., * * * a great deal of bracing is required to prevent failure by buckling." And, on page 1070,§ in discussing buttresses: "As a rule, the proportion between the lateral width and height of this pier should be between one-tenth and one-fifteenth."

A buttress of a multiple-arch dam is not to be confused with an ordinary building column, on which the maximum loading is not definitely known, and there is danger of eccentric loading, damage due to fire, etc.

Mr. Noetzli does not attempt to determine the stresses due to buckling in the buttresses, and he does not present any experimental evidence which would support his assumption. The writer has made a rough attempt to determine the safe loading of a solid buttress, having the same dimensions and the same cross-section as the buttress shown on Plate III,‖ and he does not believe that any danger of buckling exists. By adding two or three counterforts and a flange at the down-stream face, any danger from buckling can be eliminated. The writer hopes that Mr. Noetzli will establish his assumption that for high multiple-arch dams, the ordinary solid buttress with counterforts is in danger of buckling.

The disadvantage of the hollow type of buttress is that it costs considerably more, weighs less, and is more readily influenced by temperature variations and by shrinkage of the concrete due to the drying-out process. The writer has discussed this phase of the subject on page 1127§ of his paper "Stresses in Multiple-Arch Dams."

It is worth noting that the "Tirso Dam", in Italy,¶ is provided with stone masonry buttresses which, due to drying out, would shrink much less than ordinary concrete.

---

* This discussion (of the paper by Fred A. Noetzli, Assoc. M. Am. Soc. C. E., published in August, 1923, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr., Fresno, Calif.

‡ Received by the Secretary, August 21, 1923.

§ *Proceedings*, Am. Soc. C. E., August, 1923.

‖ *Loc. cit.*, p. 1083.

¶ *Engineering News-Record*, May 10, 1923, p. 820.

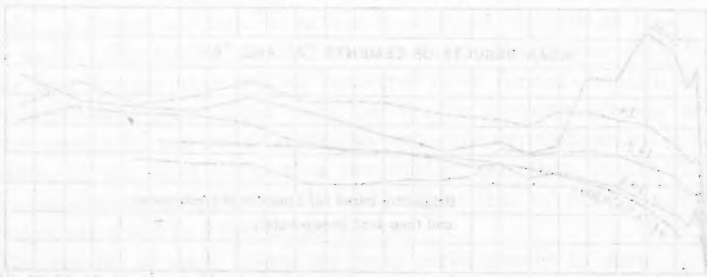
in which cement structures are to stand, and that the subject will be most thoroughly investigated.

THEODORE BELZNER,* AFFILIATE, AM. SOC. C. E. (by letter).†—The authors are to be congratulated for their paper on this important subject. The writer believes that one of the most important factors to be considered in structures exposed to sea water is a dense concrete. In most cases, the greatest deterioration occurs between tidal levels, and is doubtless caused by the sea water entering the minute pores of the concrete. Freezing and thawing result in further disintegration, which is augmented by abrasion and impact from floating objects, etc. The concrete should be protected by sheathing, or otherwise, especially where subject to damage.

Another important factor to be considered in mixing concrete (other things being equal), is the water content. The writer believes that plastic concrete gives the best results, that is, the concrete should have the least quantity of water that will permit reasonably easy handling.

* Insp. of Steel, in Chg., Housing Station No. 15, Dept. of Plant and Structures, City of New York, New York, N. Y.

† Received by the Secretary, August 20, 1923.



IMPROVED TYPE OF MULTIPLE-ARCH DAMS

Discussion*

By Messrs. B. F. Jakobsen, Victor H. Cochrane, and William Cain.

B. F. Jakobsen,† M. Am. Soc. C. E. (by letter.)‡—The author deserves praise for attempting to extend the field of the multiple-arch dam; and the writer agrees that the gravity dam is not deserving of the great confidence it generally inspires, and that its factor of safety is always much less than that of a well-designed multiple-arch or single-arch dam.

The author assumes *a priori* that it has been the danger of buckling of the buttresses, which in the past has limited the height of multiple-arch dams. On page 1068,§ he states: "For dams higher than 125 ft., * * * a great deal of bracing is required to prevent failure by buckling." And, on page 1070,§ in discussing buttresses: "As a rule, the proportion between the lateral width and height of this pier should be between one-tenth and one-fifteenth."

A buttress of a multiple-arch dam is not to be confused with an ordinary building column, on which the maximum loading is not definitely known, and there is danger of eccentric loading, damage due to fire, etc.

Mr. Noetzli does not attempt to determine the stresses due to buckling in the buttresses, and he does not present any experimental evidence which would support his assumption. The writer has made a rough attempt to determine the safe loading of a solid buttress, having the same dimensions and the same cross-section as the buttress shown on Plate III,‖ and he does not believe that any danger of buckling exists. By adding two or three counterforts and a flange at the down-stream face, any danger from buckling can be eliminated. The writer hopes that Mr. Noetzli will establish his assumption that for high multiple-arch dams, the ordinary solid buttress with counterforts is in danger of buckling.

The disadvantage of the hollow type of buttress is that it costs considerably more, weighs less, and is more readily influenced by temperature variations and by shrinkage of the concrete due to the drying-out process. The writer has discussed this phase of the subject on page 1127§ of his paper "Stresses in Multiple-Arch Dams."

It is worth noting that the "Tirso Dam", in Italy,¶ is provided with stone masonry buttresses which, due to drying out, would shrink much less than ordinary concrete.

---

* This discussion (of the paper by Fred A. Noetzli, Assoc. M. Am. Soc. C. E., published in August, 1923, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr., Fresno, Calif.

‡ Received by the Secretary, August 21, 1923.

§ *Proceedings*, Am. Soc. C. E., August, 1923.

‖ *Loc. cit.*, p. 1083.

¶ *Engineering News-Record*, May 10, 1923, p. 820.

The writer does not agree with the author's method of determining the stresses in the buttresses, but as he has fully discussed that phase of the subject in his paper, referred to previously, it would be without purpose to repeat it.

VICTOR H. COCHRANE,* M. AM. SOC. C. E. (by letter).†—This excellent paper is of particular interest to the writer because he has been engaged for some time in developing designs for multiple-arch dams similar to those described by the author. There is no doubt as to the superiority of what Mr. Noetzli calls the "improved type" of multiple-arch dam, and the wonder is that this design was not evolved years ago, especially as it does away with what heretofore have been regarded as objectionable features in dams of this kind. The author writes clearly and convincingly, and this paper, like his previous ones on this subject, will no doubt prove to be of great value to the Profession.

Some of the objections to multiple-arch dams as heretofore built are:

1.—The stability of the buttresses is dependent on the more or less flimsy bracing used, and, in case some of this bracing should be destroyed accidentally, the structure would be unsafe;

2.—The accidental failure of an arch or a buttress might cause the entire structure to fail, inasmuch as the expedients used to take care of unbalanced thrust are inadequate;

3.—Leakage through the thin arches may be sufficient to rust the reinforcing steel or cause injury to the concrete by frost.

On page 1066‡ the author disposes of the last-named objection, and the others are met, at least partly, by his improved design. The buttresses in the Horseshoe Dam are too slender to insure entire safety in case of failure of an arch or buttress. However, this contingency seems to be so remote that it might well be disregarded if it were not for the fact that it is practicable to secure a greater margin of safety without additional expense, as will be shown.

The writer, in his designs, has gone a step farther than the author, and, instead of using "buttress piers" like those in the Horseshoe Dam, builds cellular piers, of such width that they can hardly be called buttresses, and the cross-walls are made substantial enough to carry their share of the load of water and concrete. Fig. 13 shows a structure designed about a year ago, the general details of which are given in Fig. 14. As there was some apprehension that malicious persons might try to destroy this dam, two wide abutment piers were put in, so as to divide the arches into three equal groups, with the object of limiting the possible damage. This device is frequently used in bridge work. This design differs from that of the Horseshoe Dam in other respects: The arch barrel is finished at the top with a simple wall at right angles to the axis of the arch; the curvature of the intrados is constant from top to bottom of the arches; the sides of the piers are vertical, instead of battered; and the top of the pier is provided with a simple slab with fillets,

* Cons. Engr., Tulsa, Okla.
† Received by the Secretary, August 25, 1923.
‡ *Proceedings*, Am. Soc. C. E., August, 1923.

instead of the flat arch shown on Plate III.* The author's recent design, described on page 1090,† appears to be the same as that shown in Figs. 13 and 14, in respect of the vertical pier faces, constant intrados curvature, and the flat slab between the adjacent ends of arches. These details are considerably better than the corresponding features of the Horseshoe Dam.

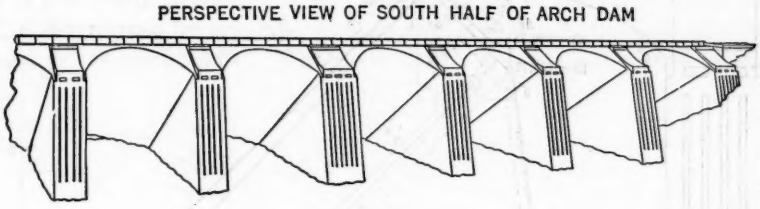


FIG. 13.

The alternative buttress pier with divergent walls (Fig. 1‡) appears to have been designed to take care of the unbalanced thrust in case of failure of an adjacent arch. As such a pier would involve complicated formwork, on account of the varying span of the intrados, a wide pier with vertical faces, such as that shown in Fig. 13, is preferable. The author, on page 1070,† seems to have reached the same conclusion. His remarks concerning the small arch on the up-stream side of the buttress do not seem to be correct. Grouping the statements on pages 1069† and 1070 and changing the arrangement slightly, they read as follows:

"It is of advantage to make the radius and the angle of opening of this small arch such that the resultant of the thrusts from the large arch and the small one is almost parallel to the opposite buttress wall. The small buttress wall is constructed preferably with a horizontal tie, so that, in case of failure of one large arch, this tie would carry the unbalanced thrust of the small arch, thus preventing it from breaking. The unbalanced thrust of the other large arch would appear to be cared for with safety".

It is evident that, if the horizontal tie takes the horizontal thrust of the small arch at one end, it will do so at the other end, and there will be no thrust from the small arch to combine with that from the large one. In its action, the small arch, either with or without the horizontal tie, does not differ from a flat slab, as far as the external stability of the pier is concerned.

A wide pier will be more economical, within certain limits, than a narrow one. There will be fewer arches and fewer piers, thus lessening the formwork and possibly the time required for completion. The quantity of concrete per linear foot of dam will be less for the arches, about the same for the pier walls, and greater for the flat slabs on top of the piers. The net result will be about the same quantity of concrete, or perhaps slightly less, and a saving in the formwork and in the placing of the concrete. Thus, it seems that, aside from any consideration of lateral stability, the use of wide piers is favored. Further-

* *Proceedings*, Am. Soc. C. E., August, 1923, p. 1083.
† *Proceedings*, Am. Soc. C. E., August, 1923.
‡ *Loc. cit.*, p. 1069.

more, the wider piers will give a desirable appearance of greater strength and reliability.

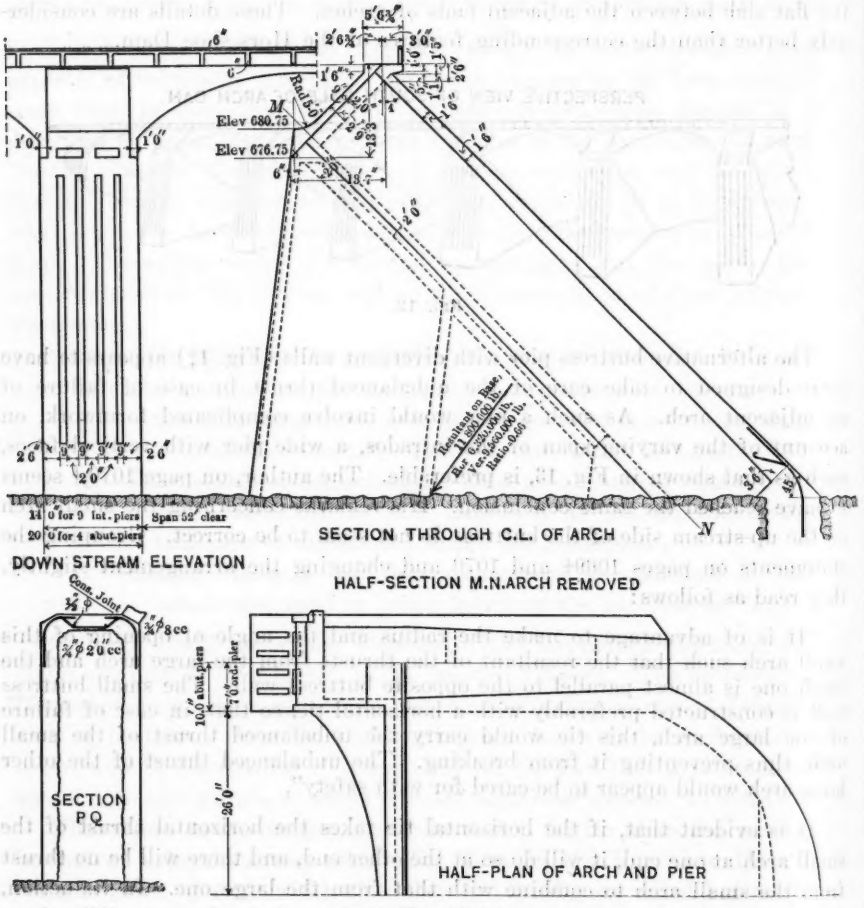


FIG. 14.

The statement frequently made—that the arch axis should be circular in a horizontal rather than in an inclined plane—is erroneous. It is possible, without changing the curvature of the intrados at any point, to fit the line of pressure much more closely than by making the arch axis circular in a horizontal plane. Near the top of the arch barrel eccentricity of the line of pressure is of no consequence, as the arch is made thicker than is required to take care of the direct stresses. From the foundation to a point near that at which the arch barrel begins to require thickening, the arch axis should fit the line of pressure as closely as practicable, and the best plan appears to be to select an arch axis of such curvature as to lie midway between the line of pressure for the section where the arch begins to require thickening and that

for a section near the foundation. Fig. 15 shows the half of an arch of about the same rise and span as the arch of the Horseshoe Dam. The line, *ABC*, is the line of pressure for the section at the beginning of the thickened part, and the line, *ADC*, is the pressure curve for a section near the base. The latter is practically circular, and may be so assumed without trial. The curve, *ABC*, is determined by graphics, as shown on page 1084*, the arch axis being assumed as the circle, *ADC*. The curve, *AEC*, drawn about midway between

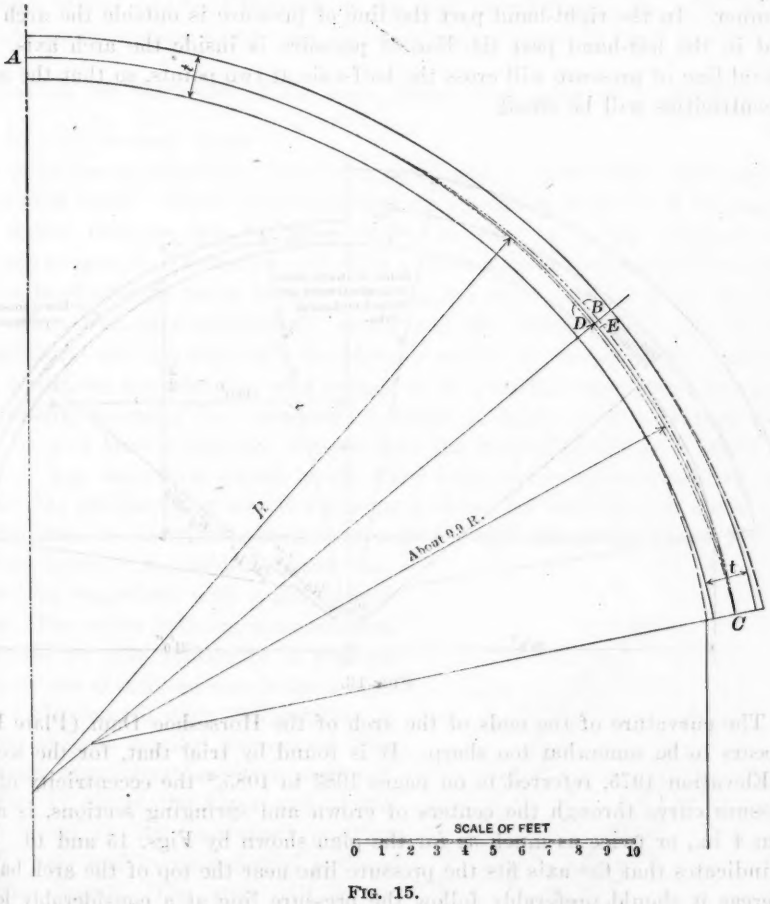


FIG. 15.

ABC and *ADC*, is the proper curve for the arch axis. It is a three-centered curve, the end radii being about nine-tenths as long as the radius for the central part. The middle two-thirds of the arch is made of uniform thickness, but it is desirable to make the two end sixths thicker toward the springing line, as indicated in Fig. 15, in order to care for the greater thrusts and the larger moments due to the eccentricity of the line of pressure and to the effects of temperature and arch shortening. It is found that a desirable

* *Proceedings*, Am. Soc. C. E., August, 1923.

amount of thickening is obtained by making the extrados a circular curve and the intrados a three-centered curve such that the thickness of the end sixths is the same on each side of the arch axis. Thus, the intrados will be a constant three-centered curve, and the radius of the extrados at any point will be equal to that of the central part of the intrados plus the thickness of the arch at the center. Of course, the ends may be thickened to any desired extent at any point by making the extrados, as well as the intrados, three-centered.

Fig. 16 shows the extreme conditions for an arch barrel planned in this manner. In the right-hand part the line of pressure is outside the arch axis, and in the left-hand part the line of pressure is inside the arch axis. The actual line of pressure will cross the half-axis at two points, so that the actual eccentricities will be small.

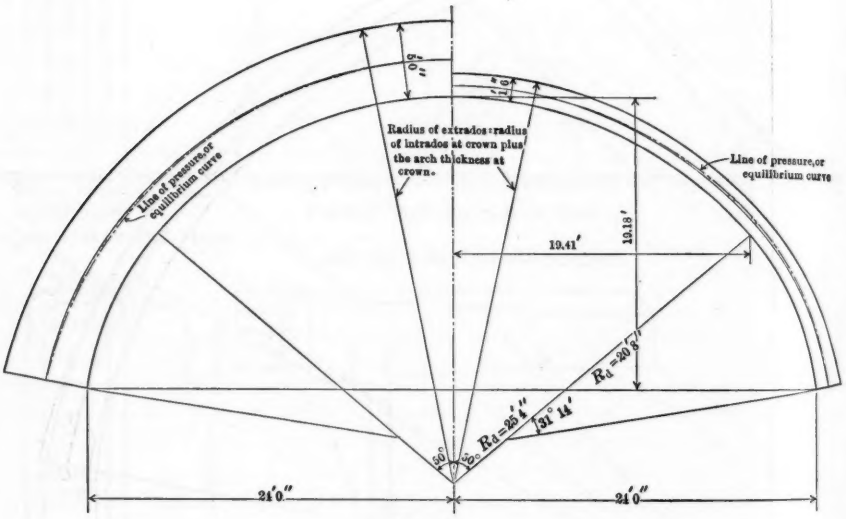


FIG. 16.

The curvature of the ends of the arch of the Horseshoe Dam (Plate III), appears to be somewhat too sharp. It is found by trial that, for the section at Elevation 1975, referred to on pages 1083 to 1085,* the eccentricity of the pressure curve through the centers of crown and springing sections, is more than 4 in., or twice as much as for the plan shown by Figs. 15 and 16. Fig. 9† indicates that the axis fits the pressure line near the top of the arch barrel, whereas it should preferably follow the pressure line at a considerably lower elevation, say, about half-way down. The section shown in Fig. 9 has a direct stress of only about 125 lb. per sq. in., so that considerable eccentricity at this point would be of no consequence.

It is inferred, from the statement on page 1086,* that the arch axis was given a sharper curvature toward the ends in order to provide an "initial eccentricity" at the springing line, thus reducing the maximum stress at that

* *Proceedings*, Am. Soc. C. E., August, 1923.
† *Loc. cit.*, p. 1084.

point. However, it should be indicated that such an eccentricity at the springing line may accompany an eccentricity of opposite character at the crown, or some other point, so that the gain at the springing line is offset by a corresponding loss elsewhere. The writer is of the opinion that it is better to fit the line of pressure as nearly as may be, in the manner previously explained.

Fig. 5* will prove to be useful in calculating temperature and arch-shortening stresses. It may be worth noting that, if the same rise and the same "cylinder stress" are used throughout the arch barrel (below the point where the minimum section ends), the arch-shortening stress is very nearly proportional to the thickness of the arch, within the limits of $\frac{t}{h}$ likely to be used in multiple-arch dams.

The author uses both an "auxiliary arch" and a "crest arch" near the top of the arch barrel. There is necessarily some ambiguity of stress at this point. The writer believes that the plan shown in Fig. 14 is the simplest that has been proposed. The arch is cut off in a plane normal to the arch barrel, and closure is effected by using an ordinary spandrel wall. The thickness required for such a wall is comparatively small, and the formwork is quite simple. Counterforts may be used over the piers or arches if desired. This construction facilitates the addition of a bridge or an over-fall apron, and provision may readily be made for a moderate increase in height at some future time.

It is well known that the stresses near the bottom of the arch barrel are more or less uncertain, owing to combined arch and cantilever action. The author has satisfactorily solved a similar problem for vertical-arch dams, but, in this case, he dismisses the matter with a single paragraph (page 1067†), perhaps because the subject is not considered so important with multiple-arch dams. The writer believes, however, that inasmuch as this ambiguity is perhaps the only one of consequence in the design of a multiple-arch dam, the question is worthy of more detailed discussion. The following approximate solution is offered.

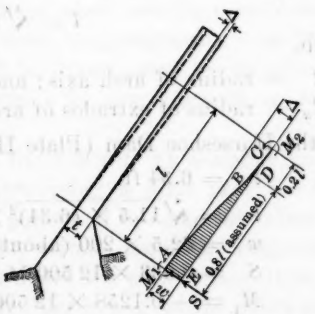


FIG. 17.

Fig. 17 shows the lower end of the arch barrel in its unstrained position (full lines) and in its deflected position (dashed lines). The arch will be restrained by the base, and the stresses in the arch will be modified throughout some length, l. In this diagram, let

Δ = the deflection of the unrestrained portion of the arch;

l = length from the fixed end at the base to the point where full arch action begins;

* *Proceedings*, Am. Soc. C. E., August, 1923, p. 1076.

† *Proceedings*, Am. Soc. C. E., August, 1923.

w = load of water and concrete, assumed for the present purpose to be constant;
t = thickness of the arch barrel, also assumed to be constant;
S = end shear, or load carried to the support;
M_1 and M_2 = moments at the lower and upper ends of the length, l, respectively;
I = moment of inertia of a portion of the arch barrel one unit wide; and
E = the modulus of elasticity of the concrete.

In the lower part of Fig. 17 the arch axis in its deflected position is represented by the line, ABC, with the point of contraflexure at B. As the deflections due to arch action are proportional to the loads on the arch, the shaded area below the line, ABC, represents the loads carried by cantilever action.

It will be sufficiently accurate to assume that the shaded area is equivalent to the area, ADE, under the parabolic curve, ABD, the distance, ED, being taken equal to eight-tenths of the length, l.

It can readily be shown that:

$$S = 0.533\ wl \quad \ldots\ldots(1)$$

$$M_1 = -\ 0.1258\ wl^2 \quad \ldots\ldots(2)$$

$$M_2 = +\ 0.0342\ wl^2 \quad \ldots\ldots(3)$$

$$\Delta = \frac{0.0113\ wl^4}{E\,I} \quad \ldots\ldots(4)$$

The author's formula for the arch deflection* is:

$$\Delta = \frac{1.56\ w\ R\ R_u}{E\ t} \quad \ldots\ldots(5)$$

Equating the second terms of Equations (4) and (5) and substituting for I its value $\frac{t^3}{12}$, gives,

$$l = \sqrt[4]{11.5\ t^2\ R\ R_u} \quad \ldots\ldots(6)$$

in which,

R = radius of arch axis; and
R_u = radius of extrados of arch.

For the Horseshoe Dam (Plate III)†:

$t = 6.34$ ft.
$l = \sqrt[4]{11.5 \times (6.34)^2 \times 25.33 \times 28.5} = 24$ ft.
$w = 62.5 \times 200$ (about) $= 12\,500$ lb.
$S = 0.533 \times 12\,500 \times 24 = 160\,000$ lb.
$M_1 = -\ 0.1258 \times 12\,500 \times (24)^2 = -\ 905\,000$ ft-lb.

The conditions at the base of the dam are shown in Fig. 18. Disregarding uplift pressure, the resultant thrust of 390 000 lb. is found to fall near the downstream edge of the base.

If the foregoing analysis is reliable, it is evident that the assumed conditions will not hold unless the base is well anchored to the bed-rock, which, to

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 12.
† *Proceedings*, Am. Soc. C. E., August, 1923, p. 1083.

the writer's knowledge, has never been done. If, however, the base is anchored to the bed-rock, the bending stresses developed at the lower end of the arch (from the moment of 905 000 ft-lb.) will be much too great; hence, it is concluded that it is not advisable to attempt to anchor the base.

With the unanchored base the up-stream edge is probably raised slightly off the bed-rock, or the concrete at the down-stream edge is crushed, or both, until the base rotates sufficiently to prevent an excessive bending moment at the lower end of the arch. It is possible, of course, that relief will occur through the cracking of the arch or the base, especially if the base is keyed into the rock at the up-stream end in such a manner that the concrete cannot move vertically a sufficient distance to provent the development of large stresses. At any rate, it appears that there must be some kind of adjustment.

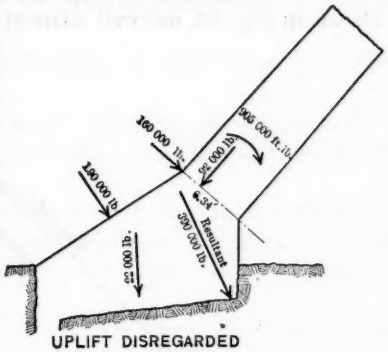


FIG. 18.

These considerations suggest that it is useless to extend the base as far up-stream as shown on Plate III, and that the cut-off wall should be placed near the down-stream edge of the base. If the base must rotate slightly, it should be dimensioned to facilitate the necessary adjustment. Furthermore, with an unanchored base, as no large bending moment can develop at the lower end of the arch, no great quantity of cantilever reinforcement is required.

It does not appear that any multiple-arch dam has developed any weakness or trouble on account of this process of adjustment at the base, and it may be that it is of no particular consequence. It is difficult to understand how the stability of the structure could be threatened in any way by what happens at this point. However, the uncertainty about what actually occurs and what dimensions and reinforcement should be used, constitutes an unsatisfactory state of affairs. The question might well be investigated by the Committee appointed by Engineering Foundation which is now engaged in studying arch dams.

It may be found advisable to overcome the difficulty by simply eliminating it altogether, which could be done by introducing a joint near the base in order to insure full arch action above that point and to make the forces acting on the base determinate. Fig. 19 shows the writer's proposed design for such a joint. It permits of full arch deflection above the point where it is placed, and provides for the ultimate closure of the lower half of the joint space by concreting under any desired conditions of loading and temperature. The saving in reinforcing steel should easily compensate for the slight additional expense of the joint. It is not necessary to extend the joint throughout the full width of the arch barrel.

The author does not mention spillway design. It seems to be entirely feasible to provide an apron on the down-stream side of a multiple-arch over-fall dam, supported either on buttresses or on the down-stream faces of

the piers, or both. Stilling pools may be used within some dams, and the multiple-arch construction makes it practicable to discharge the water over the dam so that it will not fall near any part of the foundation of the structure. The piers themselves, especially if made of considerable width, as shown in Fig. 13, are well adapted for the construction of siphon spillways.

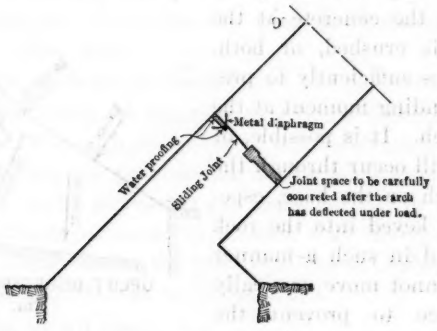

Metal diaphragm
Water proofing
Sliding Joint
Joint space to be carefully concreted after the arch has deflected under load.

FIG. 19.

The multiple-arch dam may be built with safety on shale or soft rock, provided there is adequate spillway capacity and no danger of undue or unequal settlement. With wide piers, the bases may be spread sufficiently to reduce the load on the foundation to any desired amount. The deck must be made flatter than is necessary with hard rock foundations, in order to insure safety against sliding. Furthermore, as the author remarks on page 1066*, the bed-rock must be sufficiently impervious to prevent water from percolating below the arches and starting erosion.

An incidental advantage of the multiple-arch dam with double-wall piers is that it can be readily adapted to a curved or irregular plan. It is only necessary to make the walls of the piers divergent, and to leave the arches the same as for a straight dam.

The statement on page 1067* that "the weight component of the arch, which acts on the arch, is a maximum near the crown and diminishes to zero at the abutments", might be misunderstood. Fig. 9† shows that the weight component acts with full force at the abutments. The meaning seems to be that the influence of the weight component on the pressure line becomes zero at the abutments.

The writer desires to close this discussion with the following statement, written some time ago, enumerating the advantages of the multiple-arch dam, with particular reference to a design such as that shown in Figs. 13 and 14:

1.—It is the most economical masonry dam that can be built, as it contains less concrete and much less steel than the flat-slab type of dam.

2.—It is the safest kind of dam, inasmuch as it can not be overturned and may have any desired factor of safety against sliding. No

* *Proceedings*, Am. Soc. C. E., August, 1923.
† *Loc. cit.*, p. 1084.

possible water pressure can overload the arches and piers sufficiently to cause the failure of the structure, and there is no uncertainty regarding upward pressure.

3.—It is monolithic, having no expansion joints to provide for temperature variations, and needing none.

4.—It is superior in appearance to hollow dams of other types, resembling arch bridges in outline.

5.—The narrow buttresses or walls used heretofore in hollow dams of other kinds are replaced with substantial cellular piers having the proper degree of lateral stability.

6.—The flimsy, inadequate, and expensive bracing used in current designs has been eliminated. The transverse walls in the piers, serve not only to brace the side-walls, but also to carry their share of the vertical load.

7.—The arch span may be made considerably longer than usual, thus making the sections near the top of the arch thicker and more substantial than in ordinary designs.

8.—The full rise of the arch is maintained from top to bottom, instead of being diminished toward the bottom, as heretofore, thus reducing the temperature and arch-shortening stresses.

9.—The intrados is of the same shape from top to bottom, thus simplifying the formwork, and the radii of the extrados are varied in such a manner that the arch axis conforms closely to the line of pressure.

10.—The special joint near the base insures full arch action under the maximum load and provides for the ultimate closure of the joint space.

11.—The spandrel wall at the top provides the simplest finish, causes the smallest ambiguity of stress, and facilitates the raising of the dam if desired.

WILLIAM CAIN,* M. AM. SOC. C. E. (by letter).†—The author's comparison of multiple-arch dams with gravity dams, as to volume and cost, is very instructive, and shows their comparative economy, where the site is favorable. The proposed hollow, double-wall buttresses, stiffened sufficiently to avoid buckling, are distinctly superior to the old type of thin solid walls, with a number of continuous struts between the buttresses, and extending from one side-hill to the other. Such double-wall buttresses can be used, not only for usual heights, say, up to 100 ft., but possibly up to 200 ft., if not more. The design is a distinct contribution, and it has been explained by the author so fully and clearly and its advantages have been presented so convincingly, that it does not seem necessary to give additional reasons in favor of its adoption.

The writer was impressed by a paper by Mr. R. P. McIntosh‡ as to making the horizontal section of the arch surface a circle, and the normal

* Professor Emeritus of Math., Univ. of North Carolina, Chapel Hill, N. C.

† Received by the Secretary, August 27, 1923.

‡ "Fundamentals in the Design of a Multiple-Arch Dam", *Engineering News-Record*, September 4, 1919, p. 455.

section an ellipse; or, at least, using approximations to such curves. The normal section of the arch barrel should be of such a shape that the line of pressures will lie near the line of centers of the cross-sections, and the three-centered arch, used by Mr. Noetzli, practically effects this desideratum. It will be observed from Plate III*, that most of the normal section is circular, having the larger radius, and the remainder is circular, with the smaller radius. The section is thus approximately elliptical, and a horizontal section is nearly circular, as seems desirable.

The elementary arch used for analysis is contained between two planes, 1 ft. apart, both planes being normal to the generators of the cylindrical arch barrel. The cross-section of such an arch is practically a rectangle. It is a mere assumption that such an arch can be considered as free, and that the stresses, as found from its analysis, are actually the stresses experienced in the arch barrel. The real reason for selecting such an arch is because the cross-section is practically rectangular, and, consequently, the location of the true line of pressures, for the forces supposed to act on it, can be determined by the usual procedure for an elastic arch, and the greatest stresses (tension or compression) can be readily computed. This is not true if the elementary arch is supposed to be contained between two horizontal planes, $\Delta\ h$ ft. apart; for, then, the vertical sections are not rectangles, but oblique-angled parallelograms, and a strict computation cannot be made, even for a given Δh; and the maximum unit stresses would vary with Δh.

Let Fig. 20 represent such a section at the crown. It varies from crown to springing line; and, for a central angle of 180°, it approaches a rectangle as the springing line is approached. In this diagram, ACB is a horizontal line through the center of gravity of the cross-section. The section is supposed to be subjected to a bending couple acting in a horizontal plane (perpendicular to the plane of the paper) through ACB, and, besides, a resultant pressure at C, normal to the section. With a neutral axis, DCE (perpendicular to AB), a roughly approximate computation of stresses can be made; but, if a cross-section of one-tenth the height, DE, is considered, with forces of one-tenth of those first assumed, the section is nearly a rectangle, and the maximum stresses are very different from those computed for the first section. For a rectangular section, the maximum stresses are the same for either cross-section; so that, for convenience, the arch can be taken as 1 ft. between the parallel planes; but, for the section of Fig. 20, there is no criterion for deciding the depth of section, $\Delta h = DE$; and the stresses will vary with the section assumed. Therefore, it seems to be useless to investigate a horizontal arch of an assumed depth, with the hope of either locating the true line of pressures or of finding the internal stresses. Nevertheless, a trial line of pressures can be drawn for such an arch of small depth, and its nearness to the arch axis may indicate the stability of the arch or the desirability of changing its shape. Besides, for the vertical arch dam, horizontal sections were taken, and they may be equally valid for the inclined

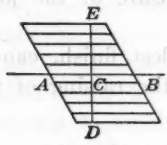


FIG. 20.

* *Proceedings*, Am. Soc. C. E., August, 1923, p. 1083.

arch barrel, as far as the path of the stresses is concerned, although such stresses cannot be computed.

Again, certain results are entirely different from those pertaining to the use of normal sections, so that the latter may not necessarily lead to true solutions, although, for the reasons given, one is compelled to use them. Therefore, it seems to be worth while to analyze, as far as may be, the horizontal elementary arch.

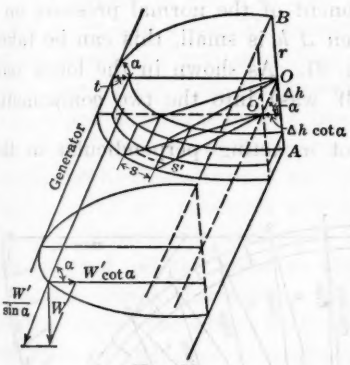


FIG. 21.

Let it be supposed that the horizontal section of the extrados of an inclined cylinder is a circle having a radius of r ft.; then the normal section will be an ellipse. Fig. 21 shows a part of a horizontal arch of vertical depth, Δh. The radius of the extrados is r and a generator of the cylinder makes the angle, α, with the horizontal radius at the crown, or with a line parallel to it. The horizontal thickness of the arch is t, then, if s is the length of the extrados of the voussoir shown, and s' is the mean of the lengths of the extrados and intrados, the area of the base of the voussoir will be $s' t$, the volume will be $s' t . \Delta h$, and its weight will be $W = 150\, s' t . \Delta h$, if the concrete of the arch weighs 150 lb. per cu. ft.

This weight acts through the center of gravity of the voussoir, or, practically, the center of s', when Δh is very small, as will be supposed. It does not act near the extrados, like W', but it will be resolved into components in the same way. Thus, the weight, W, will be resolved into the two components, $\frac{W}{\sin \alpha}$, acting parallel to a generator, and $W \cot \alpha$, acting horizontally and parallel to the crown radius, or perpendicular to the span line.

The arch is subjected to a normal pressure, p lb. per sq. ft., on the water surface; and its amount, for the voussoir, will be resolved into vertical and radial (horizontal) components according to the well-known principle: "The pressure in any direction is equal to the normal pressure on a projection perpendicular to that direction." Let C equal the horizontal component. Therefore, as the vertical projection of the water surface of a voussoir is, $s . \Delta h$, $C = p . s . \Delta h$. As Δh will be considered as very small, C acts toward the center through the mid-point of the arc, s.

The vertical component is equal to p multiplied by the horizontal projection of the water surface of a voussoir. To find the latter, consider Fig. 22 in connection with Fig. 21. In Fig. 21, the center, O' is $\Delta h \cot \alpha$ to the left of O. Hence, if, in Fig. 22, which represents a horizontal projection of the two circles, with centers, O and O', $O\,O'$ is laid off equal to $\Delta h \cot \alpha$, the required projections, a_1, a_2, . . . a_6, can be computed. As, for equal increments, Δh,

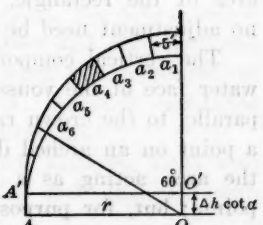


FIG. 22.

the corresponding areas, a, are the same, a varies with Δh; hence, it is most convenient to lay off $O\ O' = \cot \alpha$, determine the areas, a, and then multiply by Δh, to obtain the true area when $O\ O' = \Delta h \cot \alpha$.

The areas, a, can be computed approximately by multiplying the mean length by the mean width. The sum of such areas, for the quarter circle, should be equal to the rectangle, $A\ O' = r \cot \alpha$ (when $\Delta h = 1$), because, in sliding the circle with the center O to the position having the center O', the curvilinear area gained ($a_1 + a_2 + \ .\ .\ .$) must exactly equal the area lost, $A\ O \times O\ O'$. The vertical component of the normal pressure on a voussoir, in pounds, is $W' = p\, a\, .\, \Delta h$. When Δh is small, this can be taken as acting at the mid-point of the arc s, Fig. 21. As shown in the lower part of the diagram, this can be resolved (as W was) into the two components, $\frac{W'}{\sin \alpha}$, acting down a generator, and $W' \cot \alpha$, acting perpendicular to the span line.

The subsequent procedures can best be illustrated by an example. Thus, let Fig. 23 represent a horizontal arch of very small depth, Δh, with a semi-central angle of 60°, a horizontal thickness of 2 ft., and with $r = 28$ ft., the radius of the extrados. The generator makes the angle, $\alpha = 48°$, with the horizontal crown radius, so that the normal thickness of the arch is 1.5 ft. The lateral faces of a voussoir will be regarded as lying in vertical planes passing through O.

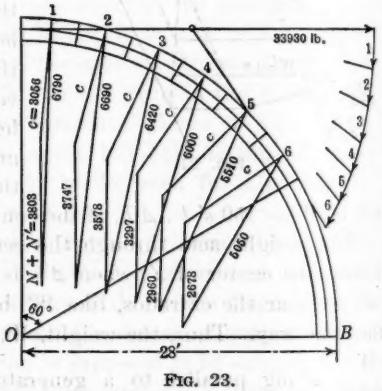


FIG. 23.

On dividing the semi-arch, Fig. 23, into six equal voussoirs, the mean length of a voussoir, measured along the central line, is $s' = 4.71$ ft.; hence its weight is,

$$W = 150\, s'\, t\, .\, \Delta h = 150 \times 4.71 \times 2\, \Delta h = \Delta h \times 1\,413 \text{ lb.}$$

Its component, parallel to the crown radius (omitting the factor, Δh), is

$$N = W \cot \alpha = 1\,413 \times 0.9 = 1\,272 \text{ lb.}$$

The construction of Fig. 22 must now be made to determine a_1, a_2, . . . The sum for the quadrant is 25.35, which is so nearly equal to the area of the rectangle, $A\ A'\ O'\ O$, Fig. 22, $= r \cot \alpha = 28 \times 0.9 = 25.2$, that no adjustment need be made.

The vertical component of the normal pressure, p lb. per sq. ft., on the water face of the voussoir, is $p\, a\, .\, \Delta h$, and its horizontal component, acting parallel to the crown radius, is $N' = p\, a \cot \alpha\, .\, \Delta h$. The normal pressure at a point on an arched dam is only equal to the water pressure at one point, the arch acting as a cantilever helping to sustain the pressure at other points; but, for purposes of illustration, suppose p to be equivalent to the head due to 10 ft. of water, then $p = 625$ lb. per sq. ft.

The successive values of N' for Voussoirs 1 to 6 were computed and added to N, the sum being given in Fig. 23. As N acts through the center of gravity of a voussoir and N' through the center of the extrados length, s, the position of the resultant, $N + N'$, was computed and laid off. From the point thus found,

$$C = p \,.\, s \,.\, \Delta h, = 625 \times 4.89 \,.\, \Delta h \text{ (omitting the } \Delta h) = 3\,056 \text{ lb.}$$

was laid off radially, and then combined with $N + N'$ (omitting the Δh), as shown on Fig. 23.

The resultants, given in Fig. 23, are laid off in order, to a smaller scale, to the right and a line of pressures passing through the centers of the joints at the crown and springing line of the 60° arch has been drawn. It passes below the center line, its maximum departure being 0.25 ft. If the arch is regarded as hingeless, the true line of pressures will lie up stream from the center line, from the crown to a certain point; and, then, it will lie on the down-stream side of the center line to the springing line. Evidently, the true line of pressures will lie so near the center line of the arch that the arch is perfectly stable. If the actual thrusts for a vertical depth, Δh, are required, all the numbers shown on Fig. 23 must be multiplied by Δh.

If the normal pressure, p, on the water surface of the arch is n times the first value, the values of C and N' will be magnified n times; and, if the thickness of the arch is unchanged, the values of N will be the same as before. The effect will be to make the resultant loads on the voussoirs more nearly radial than before and insure further the stability of the horizontal arches.

With regard to W', the vertical component of the normal pressure on a voussoir, its point of action is inside the voussoir, and it recedes from its water surface as Δh increases; so that its component, $\frac{W'}{\sin \alpha}$, parallel to a generator, likewise acts inside the arch barrel, an unknown amount, and transmits this vertical component to the base. Similarly, the weight, W, of a voussoir, is carried, parallel to a generator, directly to the base. This is a very different decomposition from that corresponding to the normal-section method, in which the whole of the normal load is supposed to be carried by arch action to the buttresses when a full arch can be drawn, and the component, $W \sin \alpha$, of the weight of a voussoir is transmitted, parallel to a generator, to the base. The results are inconsistent, and either one method must be wrong and the other right, or both may be wrong. The writer inclines to the normal-section method, as it is practical and gives results (stresses); but, as Nature doubtless works with an economy of effort, the actual transmission of loads may be of quite a complicated character and different from that corresponding to normal sections, particularly when the resistance of the arch barrel, acting as a cantilever, is considered; for such resistance relieves the arch of much of its load at the lower elevations.

As such cantilever action has not been considered, hitherto, it is well to make a few remarks concerning it, for the inclined arch dam, although a strict solution cannot, at present, be effected.

Let *FABG* (Fig. 24) represent a vertical section at the crown of a cantilever, 1 ft. thick, perpendicular to the plane of the paper. The lines shown are lettered similarly to those drawn similarly in Fig. 15 of the writer's discussion on Mr. Noetzli's tentative method* where the procedure is given in great detail. Let it be supposed that the cantilever is fixed at the base along a right section; for, if any other section is used, B. F. Jakobsen,† M. Am. Soc. C. E., has demonstrated that right sections no longer remain plane after bending, so that the usual formulas for bending stress no longer apply strictly.

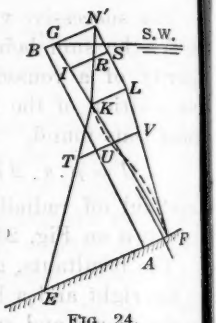


FIG. 24.

In Fig. 24, the ordinates perpendicular to *F G* represent unit pressures on this face. Ordinates of the area, *E K N' L F*, such as *V T*, *L K*, *S R*, represent water pressures at *U*, *K*, and *I*. The ordinates between *E K* and the dotted curve, *K U F*, represent unit pressures on the cantilever below *K*, all acting to the left. Above *K*, such pressures, representing the action of the arch on the cantilever, act to the right, and are represented by the ordinates, as *I R*, of the triangle, *K G N'*.

The supposed elementary arches lie between planes, 1 ft. apart, perpendicular to *F G*. The unit pressure, *p*, at the crown of the arches is represented by ordinates such as *V U* and *S I* (for the points *U* and *I*) of the area between *F L* and the curve, *F U K*, for points on *F G* below *K*, and the area, *L K G N'*, for points above *K*, all pressures acting to the left. The point, *K* (as explained in the discussion referred to), is found by trial, and is the one point on *F G* where the arch sustains the full water load.

The deflections, at right angles to *F G* (Fig. 24), of points on the cantilever, can be found easily by a construction,‡ and if (after some trials, perhaps) such deflections turn out to be the same as those computed for the elementary arches at the corresponding points, then the line, *F U K N'*, has been located properly. It may prove more convenient to rotate Fig. 24 about *F* until *F G* is vertical, and then effect the construction for a depth of water, *F G*, the forces acting on the cantilever being horizontal. This would permit the use of the analytical method of B. A. Smith, M. Am. Soc. C. E.,§ if preferred. The horizontal deflections thus found are really the deflections perpendicular to *F G* in the original inclined position, and they must be multiplied by sin α to give the true horizontal deflections.

The sole difficulty lies in finding a formula for deflection of the normal arches for various depths. Perhaps an existing formula may be used for a rough solution. No attempt has been made to derive a practically exact formula; but, granting that one can be found, then the line, *F U K N'*, can be located, the pressures, *p*, at the crown of each arch ascertained, and the bending moments at various points of the cantilever computed. There are some normal arches, near the base, which do not reach the buttresses; but

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 74.
† *Proceedings*, Am. Soc. C. E., August, 1923, p. 1108.
‡ *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 74, Fig. 16.
§ *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 2084.

their influence is nearly negligible, because, from Fig. 24, it is seen that the pressure on them is very small, nearly all the water load there being carried by the cantilever.

It seems to be customary, for a given normal arch, say, at U, to assume that the full water pressure, $V\,T$, acts at the crown, whereas only the pressure, $V\,U$, really acts there. The supposition is thus decidedly on the side of safety for the lower arches, but on the danger side for the arches above K. However, the cantilever seems to be ignored in the usual computations, although the loads on it and the bending moments are very large near the base. It may turn out that the very thin cantilever is the weak point in the design. Certainly, it seems desirable, for safety, to assume an existing formula for arch deflection, for lack of one applicable to a normal arch, and effect the construction of Fig. 24, so as to find, roughly perhaps, the bending moments in the cantilever at the base and at other points.

Similar remarks apply when the arch barrel is simply supported at the base, although, now, no tentative solution is available; but, on tilting the cantilever until $F\,G$ is vertical, as explained previously, the analytical method is applicable. The writer has given an application of this method to the Wooling Dam.*

In the final computations of stresses in the cantilever, the components, $W \sin \alpha$, of the weights of the voussoirs, acting parallel to a generator, must be finally included to find the resultant stresses on the cross-sections.

Mr. Noetzli, like the writer, has deduced a formula for the thrust due to temperature change for an arch fixed at the ends, but free from the resistances of the cantilevers to the expansion. For a rise of temperature, the cantilevers resist the outward movement of the arches by radial pulls; for a fall of temperature, the cantilevers resist the down-stream movement of the arches by radial pushes; and such pulls or pushes must be considered, in order to effect a solution. This solution has been given in great detail by the writer† for temperature change alone, with reservoir empty, but it is suggested‡ that it may be preferable to effect the construction for combined water pressure and rise of temperature, as the curved line that has to be assumed is nearly straight. The difference of corresponding pressures for the combination and for water pressure alone, will give the pressures due to temperature change only. The stresses due to shrinkage of the concrete and axial stress may be referred to those caused by a temperature fall.

It may be inferred from what precedes, that the writer does not agree with the author that an exact computation of the stresses in the arches is possible. The full solution is as complicated a problem as any presented by engineering science, and an approximate solution is all that can be expected.

With regard to the analysis for the buttresses, the method advocated by the author, and styled the "monolith" method, is the true one where there is

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), pp. 87–90.

† *Loc. cit.*, pp. 79–85.

‡ *Loc. cit.*, p. 87, Fig. 20 (b).

no dangerous tension in the arch part, wherever the section is taken. The general design of the buttress piers seems to be admirable.

The author has discussed many vital points affecting multiple-arch dams, in his interesting paper, and it is to be hoped that it will stimulate a full discussion of this subject which now appears to be of special importance to engineers.

STRESSES IN MULTIPLE-ARCH DAMS

Discussion*

By William Cain, M. Am. Soc. C. E.

William Cain,† M. Am. Soc. C. E. (by letter).‡—The author has introduced a number of interesting points for discussion. With regard to a prismatic beam, subjected to a bending moment and an axial load, the influence of shear being neglected, the statement is made, that only sections of the beam normal to the axis remain plane after the loading is applied, or oblique sections, originally plane, do not remain so under stress.

In endeavoring to prove this contention, the author correctly deduces Equations (18) and (19),§ and, from Equation (19), reaches the interesting conclusion, that, if the beam is fixed at one end along an oblique plane, the normal section, anywhere, will not remain plane after the loading is applied.

The proof is convincing, but this is not the case for the original thesis, as Equations (17)‖ and (18) only refer to a normal section and do not apply to an oblique section, $x = a\,y + b$. The section must be parallel to the transverse load, P, so that its moment is the same for any point of the section, and the shear on the section is P. The author seems to have recognized this limitation in the reasoning of page 1110,¶ in connection with Fig. 23. However, a proof can be given, by aid of Equation (18), that plane oblique sections do not remain plane after the supposed loading, as follows: In Fig. 43, let $C\,D$ be an oblique section of the prismatic beam, the equation of which is,

$$x = a\,y + b.$$

Let the co-ordinates of M, M_0, and M_1 be, respectively,

$$y = O\,E,\ x = E\,M;$$

$$y = O,\ x = O\,M_0 = b;$$

$$y_1 = -y = -O\,E,\ x_1 = E_1\,M_1.$$

After the application of the forces, P and W, the new lengths of the fibers, $E\,M$, $O\,M_0$, $E_1\,M_1$, will be supposed to be, $E\,N$, $O\,N_0$, $E_1\,N_1$, respectively. To find the latter by aid of Equation (18), one must suppose the right section through M, M_0, and M_1, successively, as Equation (18) is based on a right section, $A\,B$.

---

* This discussion (of the paper by B. F. Jakobsen, M. Am. Soc. C. E., published in August, 1923, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Prof. Emeritus of Math., Univ. of North Carolina, Chapel Hill, N. C.

‡ Received by the Secretary, August 13, 1923.

§ *Proceedings*, Am. Soc. C. E., August, 1923, pp. 1107–1108.

‖ *Loc. cit.*, p. 1107.

¶ *Proceedings*, Am. Soc. C. E., August, 1923.

(1) Suppose a right section (not drawn) passed through M. It is perpendicular to the axis, OH. For brevity, let $C = \frac{P}{I\,E}$, $D = \frac{W}{A\,E}$; then, by Equation (18),

$$E\,N = x + C\,y\left(n x - \frac{x^2}{2}\right) - D\,x$$

(2) Suppose the right section through M_0 ($y = 0$, $x = b$). By Equation (18),

$$O\,N_0 = b - D\,b.$$

(3) Conceive the right section through M_1 ($y_1 = -y$, $x = x_1 = E_1\,M_1$). By Equation (18),

$$E_1\,N_1 = x_1 - C\,y\left(n x_1 - \frac{x_1^2}{2}\right) - D\,x_1$$

On noting that, $x - b = a\,y$, the slope of $N_0\,N$ is,

$$\frac{E\,N - O\,N_0}{y} = \frac{1}{y}\left[a y + C\,y\left(n x - \frac{x^2}{2}\right) - D\,a y\right].$$

Again, noting that, $b - x_1 = x - b = a\,y$, the slope of $N_1\,N_0$ is,

$$\frac{O\,N_0 - E_1\,N_1}{y} = \frac{1}{y}\left[a y + C\,y\left(n x_1 - \frac{x_1^2}{2}\right) - D\,a y\right].$$

These two slopes can only be equal, or $N_1\,N_0\,N$ can only be a straight line, when,

$$n\,x - \frac{x^2}{2} = n\,x_1 - \frac{x_1^2}{2};$$

or,

$$n\,(x - x_1) = \frac{1}{2}\,(x^2 - x_1) = \frac{1}{2}\,(x - x_1)\,(x + x_1);$$

or, when,

$$n = \frac{1}{2}\,(x + x_1) = b.$$

As b, however, can have any value from zero to near n (so that a full oblique section can be formed), the conclusion, $n = b$, is absurd; the slopes of $N_1\,N_0$ and of $N_0\,N$ are never equal for any value of y; and, thus, it is proved that plane sections, $C\,D$, before strain, do not remain plane after strain.

The same conclusion is reached if the slopes of $N_1\,N$ and $N_0\,N$ are compared, because, on putting them equal, for $b > \frac{x}{2}$, the absurd conclusion is reached that a positive quantity is equal to a negative quantity.

The author's conclusion for the oblique section, $C\,D$, of Fig. 23, as given on page 1110,* is so important that it is worth while checking it with a slightly different procedure. Thus, in Fig. 44, extend the line of action of P to the intersection, E, with the perpendicular to $C\,D$ at O; and, at E, resolve P into components, $P \sin \alpha$, $P \cos \alpha$, perpendicular and parallel, respectively, to $C\,D$. The first component causes a uniform normal unit compression on $C\,D = \frac{P \sin \alpha}{A \sec \alpha} = \frac{P}{A} \sin \alpha \cos \alpha$, A representing the area of the right section,

* *Proceedings*, Am. Soc. C. E., August, 1923.

$A\ B$. The moment of the component, $P \cos \alpha$, about O, is, $P \cos \alpha\ z \sec \alpha = P z$. If $O\ A = O\ B = y$, therefore $O\ C = O\ D = y \sec \alpha$. Also, if $I =$ moment of inertia of $A\ B$, then the moment of inertia of $C\ D$ is $I \sec^3 \alpha$; whence the bending unit stress, normal to $C\ D$, at C and D, assuming the formula for a right section to apply, is,

$$\frac{P z\ y \sec \alpha}{I \sec^3 \alpha} = \frac{P z\ y}{I} \cos^2 \alpha.$$

The unit stress, acting perpendicular to $C\ D$, at C, is,

$$\sigma_C' = \frac{P z\ y}{I} \cos^2 \alpha + \frac{P}{A} \sin \alpha \cos \alpha;$$

at D, it is,

$$\sigma_D = -\frac{P z\ y}{I} \cos^2 \alpha + \frac{P}{A} \sin \alpha \cos \alpha.$$

If the latter expression is negative, it indicates tension.

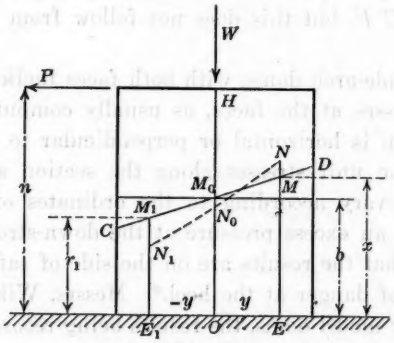


FIG. 43.

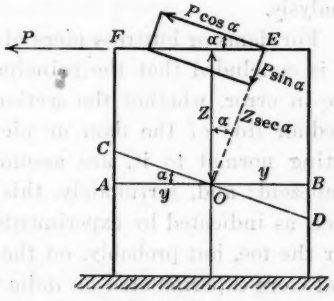


FIG. 44.

The author's Equation (22)*, $\sigma_C = \frac{\sigma_C'}{\cos^2 \alpha}$, can be derived at once, by the aid of Fig. 24,† by resolving perpendicular to $C\ F$. The unit shear along $D\ C$ (Fig. 44), $\frac{P \cos \alpha}{A \sec \alpha} = \frac{P}{A} \cos^2 \alpha$, is thus seen not to affect the value of σ_C or of σ_D, the unit stress at C or D, acting in the direction of the axis.

On dividing the stresses at C and D, normal to $C\ D$, by $\cos^2 \alpha$, the following equations are derived,

$$\sigma_C = \frac{P z\ y}{I} + \frac{P}{A} \tan \alpha,$$

$$\sigma_D = -\frac{P z\ y}{I} + \frac{P}{A} \tan \alpha,$$

which agree with the author's Equation (23).‡ In this case, $\sigma_C > \sigma_D$ (numerically), whereas the reverse is true. The formula used for bending is

* *Proceedings*, Am. Soc. C. E., August, 1923, p. 1110.
† *Loc. cit.*, p. 1107.
‡ *Loc. cit.*, p. 1110.

thus inapplicable, or only approximately applicable, to an oblique section. The term, $\frac{Pz\,y}{I}$, gives the bending stress at A or B, due to the load, P; and the true stress at C should be less than this value and that at D should be numerically greater, as the moment of P at C is less than $P\,z$, and is greater at D.

If the part of the prism above $C\,D$ (Fig. 44), is supposed to represent a portion of a dam, subjected to a load, P, C being the toe and D the heel, the usual procedure for finding the principal stresses at C and D, is equivalent to that used previously. Such a computed stress at the toe, C, is thus always too great, and at the heel, D, too small. However, in a dam, the two faces are not usually parallel, but inclined to each other. Therefore, suppose the down-stream face, $C\,F$, to rotate about C to the right. Presumably, the normal stress at C or $C\,D$, as computed, will remain too large, and thus the principal stress acting parallel to $C\,F$, as computed by usual methods, will be too large for various positions of $C\,F$, but this does not follow from the analysis.

For dams or buttress piers of multiple-arch dams, with both faces inclined, it is concluded that the principal stresses at the faces, as usually computed, are in error, whether the section taken is horizontal or perpendicular to the median line of the dam or pier. The unit stresses along the section and acting normal to it, are assumed to vary according to the ordinates of a trapezoid; and, fortunately, this gives an excess pressure at the down-stream face, as indicated by experiments, so that the results are on the side of safety for the toe, but probably, on the side of danger at the heel.* Messrs. Wilson and Gore experimented on india rubber model dams, the results being recorded in a paper† from which Fig. 45 is taken. The unit stresses in amount and direction, at numerous points in the dam, were determined from the measured distortions, and are represented by the ellipses of stress shown. The principal stresses, or those acting normally to their planes, are represented in amount and direction, by the semi-axes of the ellipses, the maximum normal stresses being given by the semi-major axes. The latter act parallel to, and very near the down-stream face. Such stresses are not generally a maximum of the principal stresses along horizontal sections, such as Sections 44 and 55. Along Sections 66, 77, and 88, in the foundation, the greatest principal stresses are in the interior, and, in fact, near the center line of the abutment, for Sections 77 and 88.

If a median line (half way between the up-stream and down-stream faces) is traced on Fig. 44, the principal stresses increase in going down stream along any section perpendicular to this median, until the down-stream face

* The remarks pertaining to wedge-shaped reinforced concrete beams, given in Appendix I of the writer's book on "Earth Pressure, Walls and Bins", may prove of interest in this connection. Reasons are given there to show that the bending unit stress, $\frac{Pzy}{I}\cos^2\alpha$, at C or D, acting perpendicular to CD, although correct for $\alpha = 0$, is too large for appreciable values of α of, say, 20° and greater, the excess increasing at a greater rate as α increases.

† *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII (1907–08), Part II.

is approached. The stresses then decrease until that face is reached, the decrease being more pronounced than for the horizontal sections.

From all that precedes—both theory and experiment—it would seem, for wedge-shaped buttresses, that the principal unit stresses at the faces are given only approximately by conventional methods, whether the section is taken horizontal or perpendicular to the median line. Further, the computed stresses at the down-stream face are doubtless in excess of the true stresses there. These remarks apply to Sections I, II, and III, of the author's Fig. 33.* The difference in results for Point *B*, according as Section II or Section III is considered, is not very large; but this difference at Point *A*, for Sections I and II, is very pronounced and alarming. Time is not afforded to study the mass of figures used in the solution and check the results; but it would be highly desirable to consider a buttress of 1 ft. constant thickness, without abutting arches, but subjected to its own weight and the water pressure on its

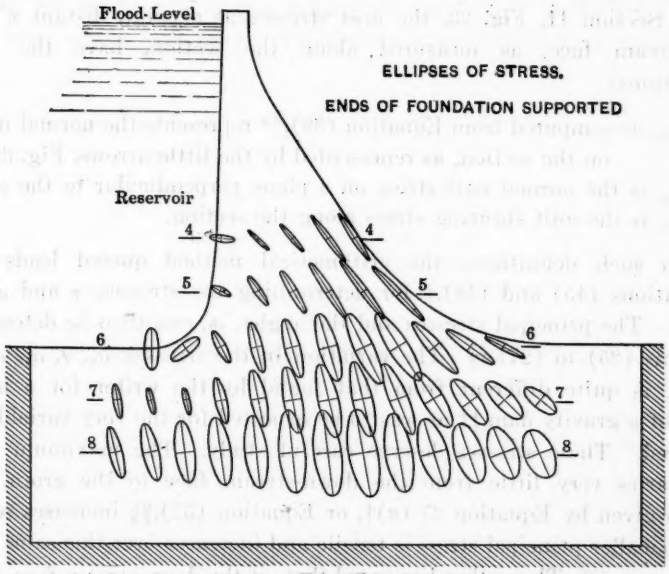


FIG. 45.

upper face, and compute the stresses at the faces for horizontal sections and for sections perpendicular to the median, and compare results. At the up-stream face, where there is a large normal pressure of the water, directly applied, the direction of the principal stress is compressive and is nearly normal to that face for usual dam designs. The fact that the author finds tension here for Section II, for the principal stress acting nearly parallel to the face, is somewhat disturbing. In treating the stability and stresses in buttresses, the author has discussed the water load. He points out a common error, and deduces correct formulas, Equations (8) and (9),† for ascertaining the point of application of the vertical components.

* *Proceedings*, Am. Soc. C. E., August, 1923, p. 1116.
† *Loc. cit.*, p. 1101.

The résumé for "Principal Stresses", with the circular diagrams of stress of Mohr and Land, will be appreciated.* It is easy to derive Equation (27)† directly from the diagrams.

The unit stresses, for heel and toe, for the three sections of a buttress with the appropriate portions of the arches, Fig. 33,‡ are given in Table 1.§ It has been pointed out that such stresses, as usually computed, are in error, whichever section is used, but that the error should be less for Section II than for the other two sections, at least, at Point *A*.

Mr. Jakobsen next undertakes the formidable task of determining the stresses at various points of Section II, Fig. 33, by aid of, what he styles, the writer's "arithmetical method". The difficulty is much increased because of the variability of the section, and doubtless most readers will take the many long formulas on faith and turn to the results, as given in Table 2‖ and Fig. 39,¶ to note the variation of the stresses.

For Section II, Fig. 33, the unit stresses at a point, distant x from the down-stream face, as measured along the section, have the following designations:

σ_x, as computed from Equation (39),** represents the normal unit stress on the section, as represented by the little arrows, Fig. 38;††

σ_y is the normal unit stress on a plane perpendicular to the section;

τ_x is the unit shearing stress along the section.

With such definitions, the arithmetical method quoted leads at once to Equations (45) and (48),† for determining the stresses, τ and σ_y, at the point, x. The principal stresses and the angles, ϕ, can then be determined by Equations (25) to (27).‡‡ The variation in the stresses, σ_y, τ, σ_1, σ_2, in the buttress, is quite different from that found by the writer for a horizontal section of a gravity dam,‡‡ as was to be expected for the very variable section considered. These stresses deserve careful study. The maximum principal stress varies very little from the down-stream face to the groin, whereas τ_{max}, as given by Equation 27 (*a*)†, or Equation (32),§§ increases somewhat, and the smaller principal stress is tensile and increases over this same interval, $x = 0$ to $x = 80$. The author has noted that, at the down-stream face, the principal stress, parallel to that face, is greater for Section III than for Section II; but, that at the up-stream face, σ_2, for Section I, is 33.4 lb. per sq. in. compression, and, for Section II, 37.6 lb. per sq. in. tension.

* The writer has found extended use for a modified form of Mohr's diagram in his book on "Earth Pressure", etc., for earth having both friction and cohesion.

† *Proceedings*, Am. Soc. C. E., August, 1923, p. 1113.

‡ *Loc. cit.*, p. 1121.

§ *Loc. cit.*, p. 1117.

‖ *Loc. cit.*, p. 1123.

¶ *Loc. cit.*, p. 1124.

** *Loc. cit.*, p. 1120.

†† *Loc. cit.*, p. 1112.

‡‡ "Stresses in Masonry Dams", *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), pp. 215–216.

§§ *Loc. cit.*, p. 1115.

The entire paper is a careful scientific discussion of a very involved subject that doubtless will be fully appreciated. The immense labor involved in the numerical computations probably would have deterred one from undertaking the solution before the introduction of the calculating machine. This mechanical aid, in this and many other subjects, will undoubtedly hasten the day of the solution of many problems in scientific engineering that, without it, would not be attempted.

As to the failure of bodies under combined stress; whether the criterion should be stress (Poisson's ratio omitted), deformation (Poisson's ratio included), or maximum shear $[0.5\,(\sigma_1 - \sigma_2)]$, engineers are somewhat at sea. The author has brought out the few facts that have been presented to date, and has suggested that when the stress corresponding to the second method gives much higher values than the stress as computed by the first method, the second criterion be accepted.

In conclusion, the writer wishes to congratulate the author on the accomplishment of a very severe task which is a decided contribution to scientific engineering.

THE RIVER AND HARBOR PROBLEMS OF THE LOWER MISSISSIPPI
A SYMPOSIUM

Discussion*

By Messrs. W. M. Black, H. B. Muckleston, H. C. Ripley, and H. F. Dunham.

W. M. Black,† M. Am. Soc. C. E. (by letter).‡—Although personal experience, or experience borrowed from books, should always be a guide in designing new engineering works, it rarely happens that conditions are so similar as to permit one work to be a copy of another. This is particularly true in the improvement of the channels over bars obstructing the entrance to a river or harbor. Each case presents conditions of its own, which must be studied; and the remedy to be applied must be that best suited to these conditions. With the exception of the rather unusual case of rock ledges, such bars are the result of the interference of two or more of the forces of Nature. These forces cannot be destroyed. When the bar is formed by the deposition of material carried down by a stream, caused by the lessening velocity of the river current on meeting the open waters of the sea, it must be expected that the material will continue to be deposited; and it will be the work of the engineer to have such material deposited where it will do the least harm, and where the cost of artificial removal, if necessary, will be a minimum and the operation of removal will not obstruct traffic. The same procedure is necessary when the bar is of drift and wave formation. Here, again, it must be expected that the supply of materials will be continued and that periodic artificial removal will be necessary at some future time. No works offering a passive resistance or a passive guidance, if such terms are permissible, will suffice permanently.

Although this is true, permanent constructions are generally necessary in order that conflicting forces may be, in a measure, harmonized and controlled, and that the active work of artificial removal may be minimized.

In making an economic design for such works, certain well-known hydraulic principles must be borne in mind. These principles are well known, but it may be wise to repeat them.

Currents are always caused by a difference in level. Once formed, they may persist for a time until their force has been exhausted, but if there is a difference in level, there will always be a flow from high to low. On an open coast, the tide wave reaches the two sides of an opening into the interior at practically the same moment, and, with the rising tide, currents will be formed toward the opening from all seaward directions. With a falling tide,

* Continued from September, 1923, *Proceedings*.

† Maj.-Gen., U. S. A. (*Retired*); Cons. Engr. (Black, McKenney & Stewart), Washington, D. C.

‡ Received by the Secretary, August 14, 1923.

the difference of head between the opening and the sea exists in all seaward directions, and the water will tend to spread toward the lower level, a tendency which is lessened, but not removed, if the water issues from the opening with a certain velocity and is directed on a definite course by reason of the inner shores. If the water, after leaving the opening is guided, by a construction concave toward the current, the centrifugal force of the flowing stream will cause the current to follow the guide closely; but this force itself causes a head to be formed against the guide and increases the tendency for the water to seek the lower level at the side. In the concave bank of a stream having a high velocity, this difference of head between the two sides of the stream is measurable, and its effect is seen in the movement of materials eroded from the concave bank across to the low side. If the head is not conserved by providing an artificial bank, the velocity and force of the current are soon dissipated.

In the case of a stream entering the sea, the force of the outgoing current is not overcome for some distance (relatively short) after the stream has passed beyond its confining banks, and the channel depth is preserved through that distance. The channel section at the shore line is generally termed the gorge. The dimensions of this section, as compared with other sections in straight reaches near the mouth, show the natural channel width and depth and form a good criterion on which to base the dimensions which should be given a desired artificial channel. The bar which has been formed where the outgoing currents have lost their velocity and dropped their burden of sediment, or where the coastal currents have been met and neutralized by the river current, is thus some distance from the shore. The deposit is usually crescent shaped, joined to the shore at both ends and crossed by several shallow and shifting channels, which are the results of the spill across the bar crest from the tidal heads and from the stream flow.

The object of the engineer is to concentrate the flow across the bar into a single channel which, to serve the volume of flow, must approximate in capacity the gorge channel. Although flow through this channel will be in alternate directions, the resultant erosive action will be seaward, because of the greater volume of the ebb flow, due to the addition of the shore run-off, and because the ebb currents prevail during lower tidal levels than the flood.

The concentration of outward flow through a single channel cannot be obtained without a conservation of head and, consequently, of current velocities on the bar. If the channel capacity as designed is insufficient, the head will be greater and erosion will take place. When the channel capacity is adjusted to the flow, the velocities will be such as to prevent new deposits. Material brought down the river will be carried farther to sea; material brought along the coast line will be deposited first along the artificial bank and in the angle between that bank and the shore, ultimately passing out seaward and working across the new entrance channel, or piling up against and passing over the inner end of the artificial shore, to be removed by dredging. It may be stated, that, although a general movement of sand in front of an entrance may be great, the resultant movement is small and is

easily cared for. In certain places on the French coast, the custom has been to excavate periodically outside the artificial bank, on the side from which the shore movement comes, and thus prevent the material from entering the channel.

The artificial banks are jetties. The channel line to be selected is the shortest line practicable from deep water inside to deep water outside, due regard being had to the necessity for guiding the outgoing currents so that they will merge with the governing coastal currents with as little interference as practicable. Although with steam navigation, the direction of the entrance channel sailing line can be located without close regard to storm wind directions, nevertheless, it is important that the wave action in the entrance channel should not be excessive and that vessels entering during storms should not have to be navigated in the wave trough. Fortunately, in America, strong coastal currents are usually wind-formed, and in guiding the outgoing channel current so as to merge with the governing coastal current, the direction to be given the entrance channel can be made such as to provide against the perils of wave action by giving a suitable length to the windward jetty. The jetties are thus located so as to protect and guide the new channel, conserving the head and volume of the outgoing currents until deep water is reached.

The jetties follow the shortest line from the shore to the inshore end of the reach in which the head and directions must be maintained. Thence, they should be parallel and at such a distance apart that proper channel dimensions will be provided. Where the shores are composed of erodible materials, care must be taken that the inshore ends of the jetties are secured against flanking and cutting during storms. As the new channel will be formed, in part at least, by erosion, the jetties must be prolonged far enough beyond the crest of the bar to provide room for a deposit of material from the crest on the outer slope of the bar. This deposit will be lessened if dredging on the crest is resorted to while the jetties are under construction and when they have reached a point where they will protect a dredged channel. The economic location of the jetties will be had when all this is accomplished.

The volume of deposits made in front of an entrance in the shore, through the long years before improvement was started, is usually very great. It is of soft and easily moved material, so that, under the action of storms, channels across it are likely to be filled and shifted rapidly when one, or more, of the natural forces acts across their line. In many cases of drift and wave bars, the deposit has actually attained stability in general form and size, annual accretions being compensated by annual erosion. The formation of an artificial channel protected by jetties across such a deposit changes conditions. The free movement past the entrance is prevented, and periodic dredging is required, but, the total annual accretion to be removed is comparatively small, with a corresponding cost of channel maintenance.

H. B. MUCKLESTON,* M. AM. SOC. C. E. (by letter).†—The writer is very glad to lend whatever weight his opinions may have to the cause of a National

* Chf. Engr., Lethbridge Northern Irrig. Dist., Lethbridge, Alberta, Canada.
† Received by the Secretary, August 22, 1923.

hydraulic laboratory for the United States. Canada is a small nation in a large territory and its hydraulic problems are great. Although it has no Mississippi with which to contend, and possibly its problems differ from those in the United States, information about natural phenomena is valuable wherever it is gathered. Canada would undoubtedly benefit by the establishment of such a laboratory.

Natural phenomena are seldom simple effects from a single cause, but are the complex resultant effects of many interacting causes. In field observations of a phenomenon it is usually impossible to separate the causes and assign their share in the resultant effect. When causes are under rigid control in the laboratory, it is possible, by varying them, one at a time, to deduce a fair theory and a reasonably accurate working formula.

Mr. Freeman has mentioned as one of the possible functions of such a laboratory the improvement of the Kutter formula. The writer believes that this formula cannot be improved, because it is fundamentally wrong in principle, but he thinks it is possible to obtain an expression that will take account of the many interacting causes in a manner not possible by the use of the Kutter formula. For instance, what effect has shape of a conduit on the mean velocity? A circle and an equilateral triangle circumscribed about it, or a square, or any polygon, have the same value of r and by the Kutter formula and by all the other formulas evolved before or since, would have the same mean velocity. Is that true in fact?

Another problem which is of vital interest in Canada and which is imperfectly understood is that regarding ice and ice formation in large hydraulic works. The theories are admittedly imperfect and little help can be obtained from practice, because an ice jam on the St. Lawrence, like a flood on the Mississippi, is an occasion for action rather than for calm study of the phenomena and their causes.

The cost of such a laboratory as outlined by Mr. Freeman is an infinitesimal part of the annual expenditure in hydraulic works in the United States and not a large part of the annual expenditure in Canada. The real question is not why it is not built now, but why it was not built long ago.

H. C. Ripley,* M. Am. Soc. C. E. (by letter).†—Some years ago—1878 to 1880—in connection with the improvement of Galveston Harbor, the writer was stationed at Port Bolivar, Tex., where there is a small creek emptying into Galveston Bay. This creek, about 2 miles long, 6 ft. wide, and 2 ft. deep, drains a marsh of limited area and discharges into the bay where the beach is all sand and the tidal oscillation is from 12 to 18 in. This little stream showed many of the characteristics of a large river emptying into a tidal sea. At its mouth there was a bar formation which deepened and shoaled with the increasing and diminishing floods caused by rains or the lack of them, and the position of the channel shifted, from one side to the other, in accordance with the direction of the tidal currents which flowed past its mouth at right angles to the direction of the stream, or closed it

* Cons. Engr., Detroit, Mich.

† Received by the Secretary, August 27, 1923.

altogether as the rains ceased and the currents and waves of the bay became predominant.

The bottom of this little stream is composed of fine sand which is formed into bars by the action of the current, just as such bars are formed in rivers. The up-stream side of the bar had a gentle slope and the down-stream side was steep. The sand, moving along with the current, was deposited on the upper slope of the bar and was washed away from the lower side to move down stream until the next bar was reached. Thus, the bar moved up stream, and the sand moved down stream, just as in a river; and all this could be plainly seen in the process, for the water was clear and the crest of each bar was only a few inches beneath the surface.

Here, then, Nature had furnished a quasi-hydraulic laboratory—although limited in its scope—where numerous experiments could be made with spurs for the removal of bars in the channel and with the numerous forms and locations of jetties for the deepening of the channel across the bar. The rising tide in the bay diminished the slope and, therefore, the velocity in the little stream, and the falling tide increased them, so that a variety of conditions occurred at frequent intervals, making many observations and experiments possible.

The writer has regretted many times that, instead of making these experiments on the real rivers and bar improvements themselves, he did not undertake them at that time, when they could have been made at small cost and, perhaps, would have saved many millions of dollars. It was with the greatest satisfaction, therefore, that he learned of Mr. Freeman's proposition for the establishment of a National hydraulic laboratory which would be available for the practical solution of numerous hydraulic problems now in dispute. It is also believed that the establishment of such a laboratory would effect a very great saving in the cost of future works of this character undertaken by the Government.

The question as to the best form and location of jetties for the deepening of bars at the entrances of rivers and harbors, has been in controversy for many years. Some engineers, of the highest authority, have contended that it is proper to use converging or parallel jetties, with dredging, and others, of equal eminence, have contended that the single curved jetty, without dredging, is the most effectual, regardless of cost. If the advocates of the latter method are correct, the cost of the improvement of most of the river and harbor entrances in the United States would be less than one-half the cost by the former methods. This question could be answered definitely by experiments in a hydraulic laboratory such as Mr. Freeman advocates. The cost of such a laboratory, compared with the saving it might effect in this one particular, is insignificant.

H. F. DUNHAM,* M. AM. SOC. C. E. (by letter).†—The author has proved his case. Using not only his own world-wide experience, but also quotations from the writings of those less experienced, he has shown that hydraulic

* Civ. and Hydr. Engr., New York City.

† Received by the Secretary, September 7, 1923.

laboratories are both helpful and indispensable. This should be accepted everywhere as a fact.

However, it is hardly advisable or wise to think of a single National hydraulic laboratory necessarily located at Washington, D. C. So well has the author presented the intimate relation between river work and laboratory results, that one is reminded of the usefulness of an industrial laboratory to a large enterprise like the erection of an enormous steel plant, although this is a mere trifle in magnitude as compared with the described river work. No one would think of establishing a steel plant on one side of a great mountain range and its laboratory on the other. The engineering problems in the United States are different from those in Germany and larger, in that American rivers are longer and carry a greater quantity of the silt burden to the sea, but they are similar in one respect, namely, the effort to invest appropriations wisely. It might well be claimed that better results could be secured with half of an appropriation, if that amount were expended in the valley to be improved instead of in the District of Columbia.

The ideal solution would be to establish a National Mississippi River hydraulic laboratory and, in its proper period of development, one in the Colorado River Valley. All lessons in the construction and use of the first would be used in the design of the second, just as the author already has drawn upon his observations in foreign countries. Once established, the continuing purpose of each laboratory should be to enable engineers to determine more effectively the behavior of that particular river. For such a difficult engineering problem, it is hard to cite a fair parallel in illustration. On a broad, rock surface with straws and shavings, a fire might be kindled and the laws of heat and air currents studied, but those same laws might not apply in a forest fire, where great branches and even tree-tops, aflame, are carried by the wind a thousand yards beyond the advancing wall of fire to start a new conflagration. After the Mississippi flood of 1913, a lady, standing at the edge of a wide, deep pit from which the tough clay and stratified rock had been wrenched out and washed away, said:

"This is where our home stood with its large garden. Once previously our house was blown away, and again another was burned; but this is worse. The tornado and the fire left the ground, but the flood takes even earth and rocks, and leaves us nothing."

The author advises full-size construction on plans derived from laboratory studies—a rather questionable course. Laws and principles may be deduced from a small model tank, but they cannot be applied to alluvial formations with the assurance that certain results would follow, even if the river valley to be studied were underlaid with an iron pan, stretching from St. Louis to the Gulf and charged with a limited variety of sand grains. The author says very little about the vast difference in material encountered when actual full-size construction is undertaken. The alluvial bank has not the same, nor even a uniform, specific gravity. The coarse sand mentioned by the author has passed along to the basement of the mighty factory where all sandstones are made and re-made. The velocity may vary greatly from that observed or calculated. Just as in a forest fire, unexpected occurrences are sure to

result from the increased dimensions, every problem is one of unstable equilibrium. It may be possible to obtain a better insight by the aid of a model of greatly increased proportions. Suppose a rather simple problem of river curvature is studied in a tank, and the controlling laws that govern flow, deposition, and shape are determined after repeated trials. Then, let those shapes be magnified accurately to a length of channel of 3 or 4 miles with similar alignment and with corresponding width and slope for a proportionately large volume of water. The channel might pass along the base of a hill of sand and loess; it should cross a formation that might boil up a thousand feet away; its banks should be of river-bank material; it should be equipped with gates, to afford close control of variable current velocities and preferably supplied with water by gravitation. Then, after an extensive test, when the water had been carefully drained away, if it were found that the current had left the channel shape little changed, the value of the "try-out" would be great. However, if it appeared at the close of this enlarged test that the channel bore no particular resemblance to the one in the laboratory from which it had been established, then the experimental value would be vastly greater.

The expense of such an effort to establish facts now unknown hardly ought to be a serious objection. A pre-determined location would be advisable, more so than the exact position of the laboratory itself. Topography should control the sites of both. No engineer having faith in laboratory work would shrink from the expense of the larger construction if he had a part of a great river channel to fashion and maintain under a contract and on his own design. Compared with the large expense of river straightening the cost of greater assurance by the method discussed would be insignificant. Microscopic examination of steel may give a fair idea of its resistance to stress, yet further tests are made. It should be remembered, too, that the expense of tests on a one-third-size model of one member of the Quebec Bridge was considered well warranted.

The author's references to unstable river conditions seem to conflict somewhat with his remarks relating to power development below St. Louis. To the writer, who has seen and been carried down the Mississippi when in flood, the idea of securing economical power therefrom seems only less feasible than the thought of deriving valuable energy from the momentum of tens of thousands of tons of moisture that pass overhead on a cloudy day. So much more credit to the one whose idea may be visionary and yet comes true.

RAILROAD TRANSPORTATION AND RAILROAD TERMINALS
A SYMPOSIUM

Discussion*

By Clement C. Williams, M. Am. Soc. C. E.

Clement C. Williams,† M. Am. Soc. C. E. (by letter).‡—Mr. Worley's paper on the "Consolidation of Railroads" presents a valuable view of the situation that will be helpful in the final solution of the problem. His close association with railway operation and with the Interstate Commerce Commission gives weight to his remarks, and the earnestness of his presentation adds to the appeal of his logic.

Mr. Worley might have laid even more emphasis on the fact that social and industrial integration has been the dominant feature of the past half century. Perhaps it might be more correct to state that social integration has been the dominating characteristic of all social evolution, for the social organism is like that of the animal or the plant, which becomes more complex and unites under one control a greater and greater diversity of functions as it ascends in the natural scale of evolution. It is only within the past half century, however, that this tendency in social institutions has become particularly manifest.

Herbert Spencer was among the first to point out the political integration from the family, through the tribe, the clan, the small nation, the large nation, and more recently the ententes and alliances of the several nations acting in concert. The development of holding corporations in industry and commerce, the amalgamation of many diverse labor unions into the American Federation of Labor and similar groups, the formation of large selling corporations, are all instances of social integration from small parts to a greater unit.

In this program of integration, the railroads are behind industry, commerce, finance, and labor, owing to the restrictive legislation of the last 30 years, the period in which the movement has been most pronounced. It is possible, therefore, for railroads to assimilate more integration at one time in the form of consolidation than would have been the case if the natural forces of economic evolution had been allowed free sway in their development. Legislation now attempts to stimulate what 30 years of legislation had sought to prevent—a circumstance that appears frequently in history.

---

* This discussion (of the papers on "Railroad Transportation and Railroad Terminals", published in September, 1923, *Proceedings*, and presented at the Annual Convention of the Society on July 11–12, 1923), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Head of Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

‡ Received by the Secretary, August 13, 1923.

The extent to which industrial integration can occur is limited only by the economies effected thereby and by man's ability to grasp the more complex situation. The magnitude and character of the operation of such a group of railways as that contemplated in the Transportation Act are such that the presidency of such a group would require a man of extraordinary powers. It would not be filled satisfactorily by a man whose only qualifications were his financial connections, nor by a Government bureaucrat. The railroad systems need not be limited to their present size because of lack of large caliber officials, however, for such men have already been active in railway circles. Private operation of railways, however, has demonstrated its superiority over Government operation so abundantly that it would be unfortunate indeed if the sound principle of consolidation of the railways should involve their unification under Government operation.

Mr. Worley states that, in negotiating the agreement of consolidation, "it should be done with charity by the strong and without avarice by the weak". The term "justice" meets both these requirements, and, happily, justice in this case is fairly definite and calculable. With the data of the Federal valuation and the past record of earnings, supplemented perhaps with a modicum of additional investigation, the true values can be acceptably estimated. The strong railroad systems, when required to surrender certain prerogatives and advantages, should be justly compensated, and the weak roads should receive only what is due them. Intangibles will constitute the chief bone of contention. Strategic terminal rights, advantageous transportation and industrial connections, and favorable line location have value, and no railroad company will willingly surrender these without adequate compensation, nor should it be expected to do so. Terminal railways have amply proved their case as the solution of city terminal problems, and, in the formation of such railways, compensation for most intangible values can be adjusted. At any rate, if a railroad company is required to yield on any point, it might properly be on the matter of intangibles, for their origin is largely social, and they belong ethically to the public quite as much as to the railroad company. The principle of "the greatest good to the greatest number with justice to the individual", should be operative against railroad companies as well as against individuals. Of course, proper compensation must be made for a fair value of such intangibles as may be surrendered and allowance made for them in the capitalization of terminal railroads.

Some railroads are chronically weak because of one or more of the following causes:

(*a*).—Poor location, either physical or economic. That is, grades and alignment are difficult, causing high operating costs, or the territory served is poor in profitable traffic. Unfavorable terminal locations and trackage rights constitute prolific causes of weakness.

(*b*).—Lack of advantageous connections, either with industries or with other carriers.

(*c*).—A top-heavy and inelastic financial structure, with excessive fixed charges, pertinent to perhaps 60% of the so-called weak roads.

(*d*).—Unwise investments, in subsidiary roads or otherwise.

(*e*).—Unsound or unscrupulous, administration.

(*f*).—Expensive terminals, with a short haul between.

In many cases only rather drastic surgery will cure these ailments. Members that are a constant drain to vigorous systems might well be amputated and junked. Lack of connections and terminal accommodations is largely a circumstance of time, and a distribution of such intangibles would enhance the value of the service to the public as well as equalize the opportunities for the railroad companies; and lines overburdened with terminals should obviously be relieved of this burden by being connected through proper grouping to terminals already well supported.

Dismemberment of existing systems seems to be deprecated as a serious obstacle to consolidation. As far as the writer is aware, very few railroads with corporate solidarity would be affected in any way even in a drastic program of consolidation. In fact, this objection is more sentimental than economic, for, with few exceptions, the dismemberment involved affects no roads having corporate unity and probably none having functional unity. Such dismemberment and re-grouping has occurred repeatedly under the machinations of capital combines without the public at large even being aware of the fact.

The writer is inclined to believe that Mr. Worley has over-emphasized what he terms the "spiritual elements of the properties and organizations", or the force of tradition. This factor is important if it exists, and no doubt it does exist in a small proportion of railway officials and employees, but the number attuned to this influence, in the writer's opinion, is inconsiderable. As the author has pointed out, the present 87 railway systems have coalesced from 2 095 railway corporations. Personal pride and private fortunes played a large part in those small local enterprizes before they began to be amalgamated into systems, but with the change in ownership and the passing of the generation instrumental in their building, the force of pride in achievement and of family tradition has ceased. The large railways are simply huge transportation machines to the employees, and the loyalty of the latter is obtainable by fair treatment quite independent of personal feelings. The writer was employed for some time by the "Road of Anthracite", a road not sufficiently extensive to be called a system, with which, because of the integrity of its history and the simplicity of its organization, an unusually fine *esprit de corps* prevailed. He was later employed by railroad companies constituting systems, roads having no singleness of history, but formed by uniting several lines and attaching numerous local roads as feeders. The employee turnover was correspondingly as large on one railroad as on the other, and there was no apparent difference in the loyalty of the workmen. In other words, the writer believes that whatever "spiritual" attachment employees may have had to original small lines has been largely dissipated in the consolidation already accomplished in grouping 2 095 railways into 87 systems. A similar state-

ment would apply to the support of the patrons of the road. A further consolidation into 19 or less systems would not affect this personal attachment seriously.

Mr. Worley apparently believes that the consolidation represented by the foregoing figures should suffice for the present. These figures do not quite indicate the entire situation, for less than 50 systems represent more than three-fourths of the mileage of the country and perhaps 90% of the revenues. A further reduction of these main groups and a gathering up of small lines as feeders would not, after all, be a violent re-alignment.

Many advantages of consolidation, in addition to those allowed by Mr. Worley, might be mentioned.

As suggested in the beginning of this discussion, this age of industry finds mighty forces in operation, contending with each other at times and co-operating at others. The day of the single-handed combat seems to be past. It is a day of combinations and mass action. It is a time of national and international labor unions instead of the old guilds and locals, of national and international industrial and commercial corporations, of industrial "blocs", farm "blocs", and other concerted and organized action. The small business is at a disadvantage in such company. If railroads are to prosper, they must be able to contend on equal terms with such forces, and consolidation into large systems will promote this end. The president of a proposed system, or, say, the presidents of all the proposed systems, would speak with a voice sufficient to obtain a hearing even in competition with the president of a National labor federation or the president of the largest financial house or steel corporation.

Under diversified control during the World War, the railway companies failed to meet the country's transportation needs, because they could not co-operate effectively. Yard tracks of one road stood idle, while another had all its yard tracks full 100 miles from its docks; on some roads, the traffic greatly exceeded the capacity, while on other competing lines it was practically normal. Unification, even with the tremendous handicap of Government operation, remedied the difficulty to a great extent. Effective co-operation would be enhanced by consolidation.

If competition of service is to be practically the only form of competition preserved, marked economies in the traffic departments could be achieved. In any event, the traffic departments would afford opportunities for considerable economy.

A more stable financial structure for the railways would result from their proper consolidation. Transportation of the scope afforded by railways is as essential to modern society as bank credit; and railroad securities should be as stable as bank securities. With an equalization of opportunity in originating traffic, as well as in transporting it between the systems, and with the change of scope of operation from local to regional, so that local disturbances would be less detrimental, this financial stability should be effected.

American industry does not need cheaper transportation service as much as it needs better and more adequate transportation, as is shown by the wide utilization of expensive auto-vehicle highway transportation. All will admit

that such consolidations as have been proposed, would afford opportunities for greatly improved service. This improvement would result largely from a more flexible and elastic car and equipment supply, more direct routing, and the elimination of car delays at terminals.

Senator Cummins called attention to one of the outstanding advantages of consolidation at the time of the passage of the Transportation Act, namely, making possible a logical and equitable rate structure that will permit the weak roads to obtain a reasonable return on investment without yielding an excessive income to the roads most favorably situated. With the centers of population and industry definitely fixed, for the most part, a grouping of the railways that will distribute advantages and handicaps to the different systems in equitable proportions, a rate structure based on cost of service, which is the only practical basis of a logical rate system, would be possible. It is probable that the strong roads would prefer to support the weaker ones directly, rather than pay excess earnings into a revolving fund for their support.

The writer agrees with Mr. Worley's suggestion that such steps of progress "to be healthy and permanent * * * must pursue natural courses" and that "progress must be by natural evolution and not by legislative force". Development of an industrial institution should not be forced ahead artificially to such an extent that it cannot articulate readily with the other agencies of industrial and social life. Relative to this matter, attention should be again called to the fact that much of the proposed progress in railroad consolidation has been inhibited for a number of years by legislative action. However, owing to the existing state of flux of transportation, due to shifting world markets and traffic routes, the present is not a propitious time to arrive at final conclusions with regard to the casting of transportation systems. On the other hand, the present situation does offer an unusually good opportunity to study the conditions on which the erection of such systems must be based.

The writer regrets that the author did not choose to discuss in a more specific way the tentative scheme of the 19 systems now being considered by the Interstate Commerce Commission, because it may be assumed that ultimately some such consolidation plan will become operative, and, from Mr. Worley's associations with railroad affairs and with the Commission, his views would have been of particular value. This matter is of such importance that it should receive careful examination and criticism. In fact, only by the aid of such criticism will the Commission be able to determine the best solution of the problem.

It is all the more imperative that this aspect of the question should receive the attention of railway engineers and officials, because of several rather fundamental and important defects and inadequacies in the proposed groupings. A somewhat bolder grouping into ultimately a smaller number of systems than 19 will be necessary, if competition is to be effectively preserved and traffic as well as financial advantages secured. In a paper before the Kansas Engineering Society in January, 1915, the writer briefly presented some of the advantages of a comprehensive unified grouping of American railways, and,

at that time, suggested 14 systems. He believes now that even that number should be reduced, perhaps to 10, rather than made larger. Mr. John E. Oldham, of Boston, Mass., Chairman of the Railway Securities Committee of the Investment Bankers Association, has published* a plan of merit, founded primarily on financial considerations, in which he proposed 13 systems. Mr. Hale Holden's proposal, before the Commission, of 4 systems for Western territory has many elements of stability and effectiveness, as compared with the 7 in the Commission's plan, and probably not more than 6 systems should comprise the railways of the Eastern and Southeastern groups. Moreover, in the writer's opinion, sufficient attention was not given to competition in north and south lines, nor to north and south traffic in general, in formulating the proposed groups.

Possibly the entire idea of competition should be abandoned, and the railways grouped territorially, as was done in England in 1921. This, of course, would require amendment of the Transportation Act. Congress preserved the principle of competition in the Act as the most effective and economical means of insuring adequate service. Whether it is better to retain this automatic but expensive regulator, or to depend on Government regulation of service, as is done by municipalities in the case of their monopolistic utilities, is a question worthy of debate. This extension of Government regulation would be nominal, for such regulation exists practically at the present time.

The obvious problem remaining to be solved in the program of consolidation is to make the project financially attractive to the railroad companies, particularly the stronger ones. Although considerable activity in the way of voluntary consolidation has occurred, it is not clear that some of the more sporadic instances of consolidation will serve to advance the ultimate program. Originally, it was expected that the provision requiring one-half the earnings in excess of 6% of the value of the property to be "recoverable by the Commission for the purpose of maintaining a general railroad contingent fund", virtually for the benefit of weak roads, would make consolidation attractive, because the roads would naturally prefer to share excess earnings with their own dependent feeders than to pay such excess into a revolving fund for weak roads in general. Results thus far show this to be only partly successful.

A liberal policy in the administration of the Contingent Fund will be desirable. Congress might even think it proper to replenish this fund by direct appropriation, with good results. Congress has seen fit to subsidize directly some industries that were obviously needed, and has seldom failed to subsidize infant (!) industries by tariffs; hence, when the importance to the nation of adequate transportation is considered, a subsidy to railways in poor territory that are necessary would not be inappropriate. However, it is probable that such a measure will not be necessary. The provisions of the Act, which contemplate a return on the value of the properties used and useful in the service, should be adequate to meet the situation. The clause, "The bonds at par of the corporation which is to become the owner of the consolidated properties, together with the outstanding capital stock at par of

* Circulated by the Chamber of Commerce of the United States.

such corporation, shall not exceed the value of the consolidated properties as determined by the Commission", should be considered as permissive as well as restrictive. A recognition of the true values of the properties involved—intangible as well as physical—will form a basis for a solution of the problem. If the railway companies can foresee adequate returns on a really fair valuation of all properties used and useful, they will doubtless enter into the scheme of consolidation with alacrity.

The question of railway groupings is of pre-eminent importance and should be given careful and critical study and full investigation and discussion. Mr. Worley's paper presents an illuminating statement of some of the general principles and difficulties, and it is to be hoped that at no distant date he may find time to make a further contribution of a more applicatory nature.

MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

HENRY EDWARD COANE, M. Am. Soc. C. E.*

DIED APRIL 29, 1923

Henry Edward Coane, the son of John Montgomery Coane and Emma A. H. Coane, was born at Ballarat, Victoria, Australia, on December 19, 1877. He received his education at the Brighton Grammar School, near Melbourne, Victoria, and was matriculated at Melbourne University. In 1892, he entered his father's office as an articled pupil, and carried on his engineering studies there and in the Melbourne Technical Schools, qualifying as a licensed Land Surveyor, Mining Surveyor, and Municipal and Hydraulic Engineer.

His father, who was then engaged in a general practice in Australia and the islands of the Central Pacific, soon admitted him to partnership. Their practice was in Australia, principally in the States of Victoria and New South Wales, where they built several water-works systems. They also constructed railways in the Maitland Coal Mining District of New South Wales.

At the same time, they served as Engineers to the Pacific Phosphate Company, of London, England, and built extensive appliances at Ocean Island and Nauru for the exploitation of the great phosphate deposits on those islands. In 1908, they opened the phosphate deposits of the Island of Makatea, Paumotu Archipelago, for the Compagnie Française des Phosphates de l'Océanie, which are now being worked by the Anglo-French Phosphate Company, of Paris and London.

When the works of the Pacific Phosphate Company were purchased by Great Britain, the Commonwealth of Australia, and the Dominion of New Zealand, and placed under the British Phosphate Commission, the firm continued the construction of improved working appliances, and, at the time of his death, Mr. Coane was preparing a mechanical system of loading with a capacity of 500 tons per hour. His death, however, intervened before the work was begun.

For considerable time before his death, his firm acted as Consulting Hydraulic Engineers to the Victorian Electricity Commission, which is building a large combined system of electrical generation and distribution, making use of extensive brown coal deposits and hydraulic power in the mountainous districts of Northeastern Victoria.

Mr. Coane was elected an Associate Member of the American Society of Civil Engineers on August 31, 1915, and a Member on April 26, 1921.

* Memoir prepared by J. M. Coane, M. Am. Soc. C. E.

ROBERT BREWSTER STANTON, M. Am. Soc. C. E.*

DIED FEBRUARY 23, 1922

Robert Brewster Stanton was born at Woodville, Miss., on August 5, 1846. His father, Robert Livingston Stanton, who was a Presbyterian minister, and a former President of Miami University, at Oxford, Ohio, was an intimate friend and adviser of President Lincoln.

Robert Brewster Stanton received his early education from tutors and at private schools. He attended Miami University from which he was graduated in 1871 with the degree of B. A.; in 1878, he received his M. A. and, later, the degree of C. E. In 1882 Wooster College conferred on him the honorary degree of M. A. In 1912, Miami University offered him the degree of Ph.D., but, owing to professional duties, he was unable to attend the Commencement exercises to receive it.

Soon after leaving college, Mr. Stanton served as an Assistant on surveys, in Indian Territory, for the Atlantic and Pacific Railroad, now the St. Louis and San Francisco Railroad, and from May, 1872, until May, 1874, he was employed as an Assistant on the preliminary and location surveys for the Cincinnati Southern Railway in Kentucky and Tennessee. He was then appointed Engineer in charge of 10 miles of the Mountain Division of this railway. His work comprised almost every kind of heavy gradation and masonry, as well as the substructure for two bridges and viaducts, and four tunnels. He continued in this position until January, 1876, when he became Resident Engineer of several residencies on the Dayton and Southeastern Railroad in Ohio, in charge of all work from first grading to finished track-laying, wooden bridges, trestles, etc.

From February, 1877, until February, 1878, Mr. Stanton was engaged as Assistant on surveys for the Kentucky Central Railroad. During this time, he was granted a three months' leave of absence to join a small party under the late Jacob Blickensderfer, M. Am. Soc. C. E. for the purpose of making a horseback exploration for the extension of the Atlantic and Pacific Railroad from Venita, Indian Territory, southwest through Texas, to join the Texas and Pacific Railroad. The distance covered, including the round trip over two lines, was about 1 200 miles.

From May to October, 1878, Mr. Stanton was employed in the office of the Cincinnati Southern Railway, and from that time until March, 1880, he served as Resident Engineer on the same road, in charge of the track-laying and erection of bridges and buildings on the Northern Division. Among other things, his duties comprised the inspection of the erection of nineteen iron bridges and viaducts and four wooden bridges and trestles.

In 1880, he was engaged as Division Engineer with the Union Pacific Railway, remaining with this Company for four years. During this time, he built the Georgetown Loop near Georgetown, Colo. After severing his connection with the Union Pacific Railroad Company in 1884, he joined the

* Memoir compiled from information on file at the Headquarters of the Society.

Engineering Corps of the Denver Circle Railway Company. In 1885, he was engaged in private practice, and from 1886 to 1888, served as General Manager of a gold mine in Idaho.

In 1889 and part of 1890, Mr. Stanton was employed by the Denver, Colorado Canyon, and Pacific Railroad Company as Chief Engineer of an expedition sent out by that Company to make a survey of the Grand Canyon of the Colorado River, from Green River in Utah to the Gulf of California. The first survey of this region was a failure, three men, including the President of the Company, having lost their lives in the attempt. Building new boats after his own design, Mr. Stanton started anew and successfully completed the survey (his party having been the first to navigate the Colorado for its entire length), and reported that it was practicable to construct a railway along the river to the Gulf of California, and operate it with hydroelectric power supplied by the river.

In 1890, he became interested in mining and continued in this field for six years. Following this period, he was employed by the Canadian Pacific Railway to examine landslides in British Columbia, and on the completion of this work, he was engaged in placer mining along the Colorado River, for the next four years. During 1902, he was employed on surveys for the Chicago, Rock Island, and Pacific Railway Company, and, in 1903, he was with the United Mining and Development Company. In 1904, Mr. Stanton visited the Island of Sumatra to examine the mines located there, and the next year continued his mining work in California, after which he became connected with the Boise City Railroad.

In 1906, he was occupied with work for the Michigan Central Railroad, and from 1907 to 1909, he was interested in mines in California, Mexico, Arizona, Red River, New Mexico, and Cuba, but, in 1910, he returned to railroad work. In 1913, he was engaged in the examination of gold mining properties in Nova Scotia.

In addition to his engineering and mining work, Mr. Stanton found time to write some interesting papers on technical subjects. His articles on the Colorado Canyon have been published in several magazines. His paper "The Alleged Journey and the Real Journey of James White on the Colorado River in 1867", appeared in *The Trail* published in Denver, Colo., in September, 1919. Some of his other works are "Land Slides of British Columbia", published by the Institution of Civil Engineers in 1896, and "Personal Memoirs of the Man" (Lincoln), which was published in *Scribner's Magazine* in July, 1920. He also prepared an extensive work on the Colorado Canyon, which has not yet been published, containing hundreds of photographs taken by him, a complete geologic history of the river, and an exhaustive study of the Spanish and early American expeditions.

Mr. Stanton was married in Denver, on December 1, 1881, to Miss Jean Oliver Moore. They had five children, four of whom are living. The last few years of his life were spent at New Canaan, Conn. He died of pneumonia on February 23, 1922, and was buried at Sewickley, Pa.

He was a member of the American Institute of Mining and Metallurgical Engineers, Institution of Civil Engineers, Institution of Mining and Metallurgy, Engineers Club, New York City, the Alpha Delta Phi and Phi Beta Kappa Fraternities, having served on the Executive Council of the former. He was also a member of the Explorers' Club and the "Ends of the Earth". A short time before his death, Mr. Stanton resigned as a corporate member of the Explorers' Club and was immediately and unanimously elected a Corresponding Member, as a token of that Club's esteem.

Mr. Stanton was elected a Member of the American Society of Civil Engineers on September 1, 1880, and served as a Director of the Society from 1894 to 1896.

CLARENCE EVERETT TAIT, M. Am. Soc. C. E.*

DIED APRIL 5, 1923.

Clarence Everett Tait, the only child of William and Mary Tait, was born at Vevay, Ind., on January 28, 1875. After completing the course at the Vevay High School, he entered the Purdue University and was graduated therefrom in 1889 with the degree of B. A. in Mechanical Engineering.

In August, 1889, Mr. Tait entered the service of the United States Department of Agriculture, Irrigation Investigations, at Cheyenne, Wyo., and drafted plates showing the design of typical irrigation structures, for the United States Government Exhibit at the Paris Exposition.

In November, 1899, he was appointed Assistant Engineer in the U. S. Irrigation Investigations, with headquarters at Cheyenne, and designed and assisted in the design of current-meter rating stations, water-stage registers, measuring weirs, rating flumes, etc. While in this position, in 1902 and 1903, he investigated and wrote a report on the storage of water in Northern Colorado, which was printed by the United States Department of Agriculture.†

In 1903, Mr. Tait was placed in charge of the work of the Central Division of the U. S. Irrigation Investigations at Cheyenne. He co-operated with the State Engineers of the Rocky Mountain States, supervised the field work for a report on the North Platte and South Platte Rivers, and designed and constructed works for the first rice irrigation experiment in Arkansas, afterward writing a report on the subject.

On account of failing health he was transferred in 1905 to Southern California and was associated with Professor Joseph N. Le Conte of the University of California, in mechnical tests on pumping plants. The results of these tests were published by the U. S. Department of Agriculture.‡

In 1906 and 1907, Mr. Tait made an investigation of conditions on the Lower Colorado River and wrote a report on "Irrigation in Imperial Valley,

* Memoir prepared by A. L. Sonderegger, M. Am. Soc. C. E. and Harry F. Blaney, Assoc. M. Am. Soc. C. E.

† *Bulletin No. 134*, U. S. Dept. of Agriculture, Office of Experiment Stations.

‡ *Bulletin No. 181*, U. S. Dept. of Agriculture, Office of Experiment Stations.

California."* In 1908 and 1909, he had charge of the work of the U. S. Irrigation Investigations in Arizona and Imperial Valley, and investigated silt problems and canal cleansing in the Southwest and on the Rio Grande in Texas. In 1910, Mr. Tait returned to Southern California where he had charge of the work of the U. S. Irrigation Investigations, with headquarters at Los Angeles, Calif., where he remained until his death.

In 1911, he wrote a report entitled "The Use of Underground Water for Irrigation at Pomona, Calif.", which was published by the United States Department of Agriculture.† In 1912, in co-operation with the State Conservation Commission, he supervised the field work in Southern California on the investigation of irrigation resources of the State, and wrote a report thereon.

In 1917, Mr. Tait's report on "Conservation and Control of Flood Water in Coachella Valley, California"‡ was published, and, in 1918, in co-operation with the State Engineer he made a report on "Utilization of Mojave River for Irrigation in Victor Valley, California".§ About this time, he was promoted to be Senior Irrigation Engineer.

Mr. Tait also had charge of an extensive silt investigation on the Colorado River and in Imperial Valley, and was writing a report on the subject, but was unable to complete it before his death.

He will be remembered as a man who won the respect of all those associated with him. In the words of one of his old friends:

"Mr. Tait stood high in his profession and was considered an authority in his line of work. Although waging a losing battle against ill health for many years he was painstaking, conscientious, and thorough in all he undertook. Only an indomitable will and splendid courage kept him at his desk. He was a man of high moral standards, even tempered, quiet, and retiring. He possessed a strong sense of justice in his dealings with others and commanded the highest respect of his associates."

Another friend states:

"Mr. Tait was a familiar figure among the engineers of Southern California, and will be greatly missed not only for his friendliness and cordiality, but also for his professional knowledge and achievements. To those who called on him for information and advice, and they were many indeed, he always was a cheerful and generous giver. His judgment and experience in matters of irrigation engineering were highly appreciated by all interested in his line of work. His good work still lives on with his memory as of a man that filled his post."

He is survived by his widow, who before her marriage was Miss Frances Walton, and by his daughter, Jean Tait.

Mr. Tait was elected a Member of the American Society of Civil Engineers on July 6, 1920, and was an active member of the Los Angeles Section of the Society. On January 19, 1923, he was appointed a member of the Executive Committee of the Irrigation Engineering Division of the Society.

---

* U. S. Senate Doc. No. 246.

† *Bulletin No. 236*, U. S. Dept. of Agriculture, Office of Experiment Stations.

‡ *Bulletin No. 4*, California State Dept. of Engineering.

§ *Bulletin No. 5*, California State Dept. of Engineering.